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Property account simulation using a stochastic matrix

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Property account simulation using a stochastic matrix

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INTRODUCTION

The prognostication of mortality characteristics of industrial property, commonly called life analysis and estimation, is important because it is critical input to the decision making processes of industrial firms and various government agencies. Major areas of public utility regulation, income tax calculation, and corporate economic decisions are all affected by these characteristics.

Mortality characteristics usually have been ascertained by one of three techniques; the actuarial methods, the turnover methods, and the forecast methods (1). The majority of the research has been associated with the actuarial methods and has taken two distinct paths. One path the investigators have trod is finding mortality laws that better describe the retirement pattern of property. A mortality law may be described as a probability density function f, where $f(x)$ is the percentage of units or dollars placed which are retired in the x age interval. This is well illustrated by the works of Winfrey and Kurtz (20), Winfrey (18), Couch (4), Kimball (11), Cowles (5), and Henderson (8). The second path represents the work accomplished in finding and/or applying better techniques in which the mortality laws of industrial property are used. Research works of Winfrey (18), Nichols (14), Lamp (12), and White (17) are illustrative of these investigations.

Presently questions are being raised as to how the various techniques and mortality laws of life analysis compare with one another. Further questions are being asked about the accuracy and sensitivity of the life analysis techniques. The probe was begun in these areas with the work of Henderson (9). Other questions are being asked concerning the behavior of the depreciation reserve (13, p. 229) since the Internal Revenue Service and state and federal public utility commissions regard the depreciation reserve as a value to be considered in the process of making tax and rate regulation decisions. Pollock's (16) work was a beginning which illustrated the importance of depreciation reserve in income tax analysis.

The Desirability of Simulating Property Accounts

In order to analyze the various life analysis techniques, a property account must be realistically constructed following a given mortality law and a given set of economic conditions. Once constructed, several general types of investigations may take place. First, the various life analysis techniques may be compared to one another; second, the life analysis procedures may be compared to a known standard; and third, these various methods may be examined for their sensitivity to detect change in the mortality characteristics. The impact of such an analysis would be observed by more accurate income tax calculations, subsequently better net income and balance sheet statements, and more effective corporate economic decisions and corporate models.

To be able to understand the behavior of the depreciation reserve also requires a simulated property account which realistically follows a given mortality law. Within the past few years a proposition (16, p. 34):

> "In its present form the reserve ratio test gives an accurate determination of the degree of conformity between tax lives and actual lives...."

reflects the current thinking of the importance of the reserve ratio in the decision making process of the Internal Revnue Service. Public utility commissions have long looked at the depreciation reserve account and based numerous decisions on rate base, rate increase, and general regulation on what they found. To understand the behavior of the depreciation reserve account and apply such knowledge in a decision making process would require an analysis of the reserve variations and the range of values which they cover.

The previous paragraphs have stated in general terms the desirability of understanding the behavior of the depreciation reserve and the evaluation of life estimation techniques. To understand what precisely must be accomplished, a closer look must be taken at what the investigations will involve.

Consider first the area of depreciation reserve behavior analysis. In order to define the depreciation reserve account two specifications must be given. First the depreciation method, which supplies the depreciation base from information generated by the property account, must be described. Second, the depreciation rate, which is multiplied by the base to determine the depreciation charge, must also be given.

It is noted that there are three representations of the depreciation reserve. These values will be referred to as pure, theoretical, and simulated depreciation reserve. The pure depreciation reserve in any year would be calculated from a deterministic set of property retirement experiences applied to a suitably defined property account and reserve account. It represents the value of reserve if all past property placements have conformed exactly to the given mortality law.

The theoretical reserve is a calculation based on the amount of property in service in a particular year, on the age of the property, and on the depreciation method and procedure. Let the depreciation charge be calculated by the straight line method, average life procedure. If $P(x,y)$ represents the percentage or dollar amount of property in service with age x in year y then the theoretical reserve in year y is calculated by P(x,y)(1-Expect(x)/ASL) where x=1,2,3,...,n. (13, p. 227) Expect(x) denotes the expectancy at age x where Execpt(x) = $\sum$kf(k+x)f(k+x) where k = 1,2,3,..,n. ASL = average service life of the property and is calculated by ASL = $\sum$ xf(x) where x = 1,2,3,...,n. This method assumes that the property retirements in the future will exactly correspond to the given mortality law. Thus the technique yields an exact value of depreciation reserve which is sufficient, given the present property in service will follow exactly the mortality law. If the previous retirement experience before the year in question exactly conforms to the stated mortality law, then the pure and theoretical reserve will be the same.

The simulated value of depreciation reserve is calculated dynamically year by year and conforms to the past and present retirement experience of the property account. Like the theoretical reserve, if the simulated values correspond exactly to the specified mortality law the values would be equal to the pure reserve calculation. It would also seem reasonable that the central tendency of either simulated or theoretical depreciation reserve in any one year would be close to the pure reserve value.

As a beginning to understand the behavior of the depreciation reserve accounts, comparisons among the pure, the central tendency of the theoretical, and the central tendency of the simulated depreciation reserves might be useful. An analysis of the variations about the central tendencies might also reveal the type and extent of decisions which could be made.

In the previous comments it was assumed for comparative purposes that the specifications of the property account and the depreciation reserve account as well as the characteristics of the mortality law were held constant. The property account specifications are two-fold. Whether an account is open or closed must be given (16, p. 21). Secondly, if the account is open ended then the growth characteristics must be described. Characteristics of a mortality law may be described as all the parameters which specify the probability density function of retirements f(x). Major variations in value of f(x) will be referred to as trends whereas minor variations will be called random fluctuations. It is recognized that it is important to ascertain the differences in behavior of the depreciation reserve under a variety of conditions. These comparisons must be made in order to understand the effects of changes in mortality law characteristics, in depreciation reserve account specifications, and in property account growth patterns on the depreciation reserve. Only by broad based research into these areas will conclusions be forthcoming about the effectiveness of the depreciation reserve as a factor in specific decision making processes.

The evaluation of life analysis techniques has already begun with the investigations of Henderson. His objective was "to select the functions most commonly used in actuarial life analysis and to test them empirically by comparing their ability to fit simulated and actual mortality data" (9, p. 32). Another area of investigation would be to ascertain the accuracy of various mortality fitting techniques. Still another would be to determine the effect of growth of the property account on the accuracy of the methods of life estimation determinations and/or the goodness of fit of the commonly used mortality laws. The sensitivity of the life estimation methods yields yet another fruitful field for investigation. The primary question being, can the estimation procedure ascertain an actual change in growth or change in mortality characteristics and ignore random fluctuations. Well constructed investigations in the aforementioned areas will lead the way not only toward better corporate models and economic decisions, but also towards more effective decision making processes of our state and federal agencies whose actions have such a tremendous impact on corporations.

Review of Property Account Simulators

In order to discuss the features of property retirement simulators which have been constructed by Pollock, Lamp, Henderson, and White, a discussion of the methods of classification of property retirement generators is required.

Retirement experience simulators may be classed in two areas; deterministic and non-deterministic. If the procedure for generation of retirement experience follows the mortality dispersion exactly for each placement of property, then such a technique is called deterministic simulation. It is recognized that in actuality the property retirements do not exactly follow a smooth mortality law. If economic conditions such as major trends and random fluctuations are part of the simulation then the property account generator is referred to as non-deterministic simulation. Thus, retirements in the k age interval of a vintage placement are modeled deterministically as $f(k)$ whereas they would be described non-deterministically as $f(k) + q$ where q is the deviation or irregularity from the expected mortality law $f(k)$.

The first major property account generator built was Pollock's. His retirement experience simulator was deterministic. The property account could be specified as either closed or open ended with an option of a non-stochastic growth factor in the latter. The reserve account could be calculated using one of three depreciation methods: straight line, sum-of-the-year-digits, or double declining balance. Since such a simulator did not attempt to model irregularities from the specified mortality law it could not be very representative of an actual property account.

Following Pollock was the work of Lamp. His research required the use of a non-deterministic simulator, but not the application to the construction of a property account as an input to make a depreciation reserve account. To be able to generate deviations from his specified mortality law, Lamp used a simple Monte Carlo technique by which he sampled from his given age-life mortality distribution (4, p. 136). The result was a new mortality dispersion h(k) which closely resembled the parent mortality distribution f(k). Obviously any irregularity q was equal to f(k) - h(k) and thus gave an approximation of the observed behavior of actual property accounts. Lamp stated in one of his conclusions, "The variance of distribution of retirement ratios for a given age interval decreases as the vintage group size increases" (12, p. 131). This, of course, would be expected since q, the deviation, is inversely proportional to the vintage group size or sample size (15, p. 61).

Part of Henderson's research required a set of simulated property retirement experiences which smoothly followed a specified mortality distribution and also retirements which were more irregular. His method of achieving this was to apply Monte Carlo simulation, as did Lamp, to a given mortality distribution. Henderson stated, "The size of the large sample was selected such as to make the resulting life table relatively smooth.... The size of the small sample was subjectively chosen with the objective in mind of simulating commonly encountered irregular data" (9, p. 81).

White's work was stimulated by the same objectives as this research (17, p. 1). That is, a property retirement experience generator was needed to examine the validity of life analysis techniques in use today as well as ascertain the behavior of the depreciation reserve account under a variety of conditions. His retirement generator was based on the same Monte Carlo technique which Lamp and Henderson used, but the results were applied to an open end account with an option of using a stochastic growth factor. The mortality simulation was more elegant since it was possible to change mortality characteristics and property account growth factors at will. The Monte Carlo simulation utilized not only placement of units, but also dollars with the option of the price per unit being a stochastic value. As was the case with the retirement experience simulator of Henderson and Lamp, White's simulator of deviations, $q = f(k) - h(k)$ was dependent on the number of samples, though he could, in a limited fashion, vary his sample size, from the parent mortality law.

At this point a problem which was not solved became apparent. If the mortality law could be changed, then how would its change affect the retirement of property previously placed?

An example of this problem would be to consider a placement of property ten years ago where it is expected that the last dollar to be retired will occur at an age of fourteen and one half years. Presently a placement of property retirements follow another mortality distribution whose last dollar to be retired occurs at an age of eight and one half years. Obviously the presently placed property is retiring faster than that placed ten years ago. This may be due to a variety of reasons such as technological obsolescence, corporate policy, or excessive use. It is important in the simulation of property accounts to adjust the retirements of the older policy in light of the present mortality characteristics.

It appears obvious that the Monte Carlo technique used in the manner in which White, Lamp, and Henderson applied it has limited usefulness in generating realistic property accounts. The difficulty was suggested by both Henderson and Lamp in the fact that the deviations q from mortality law $f(k)$ were a function of sample size. This would imply that any useful conclusion would first have to prove that the selected sample size used to make irregularities would generate a realistic retirement experience whose characteristics would relate well to those observed in actual property accounts.

A second reason why the aforementioned technique works poorly is that once a placement of property has been distributed by the Monte Carlo procedure there is no provision to relate the retirement of the x year and subsequent years with conditions either external or internal to the company which start and continue in year x. This major flaw would make it extremely difficult to implant the Monte Carlo technique of property account simulation as a subsection of a larger corporate model.

Thrust of Research

The previous discussion has made clear that a better property account generator is required before any conclusive results may be obtained. It was the major emphasis of this research first to find a new concept around which a retirement experience simulator could be built and then build such a device. Once built, a series of trial runs were to be made to ascertain the effectiveness of the generator.

The evaluation of the trials would be two-fold. First the major characteristics of the generator would be analyzed and compared to existing property account simulators. Second, a cursory inspection would be made of several areas of interest. The inspection would consist of graphically illustrating various results of the trials and conducting appropriate statistical tests on the information generated. It would also include any calculations which might be conceptually interesting. Though the evaluation might contain several limited conclusions as a result of the cursory inspections, it must be remembered that the major thrust was to find, build, and test a property account generator which has the capability of successfully giving information concerning the areas of life analysis and depreciation reserve described in the previous sections.

THEORETICAL BASIS AND CONSTRUCTION OF THE PROPERTY ACCOUNT GENERATOR

A stochastic matrix, with non-negative elements and unit row sums, P , (7, p. 375) which controls the retirement of property whose remaining life is represented by a position in a vector D , is the basis of the property account simulator. The discussion of the development of the matrix and its final application to a retirement experience simulator follows in the next five sections. Section one discusses the basic development and application to a vintage property account. The second section applies this development to a continuous property account. Section three states the various approximations required in order to relate the work to common usage of the practitioners in the field. The fourth section describes the effect of changing the elements in the stochastic matrix. The last section describes in detail the capabilities of the property account simulator.

Deterministic Elements of a Stochastic Matrix for a Vintage Account

Let the vector $D=[d(r), d(1), d(2), \ldots, d(n)]$ where $d(r)$ is the unit or dollar amount of property retired. Then $d(1)$ denotes the amount of property with one year of life remaining, and $d(n)$ represents the amount of property with n years of life remaining (21, p. 717). Let D'(i) and D(i) denote the property vector D at the beginning and end of period i respectively. If f(x) x=1,2,...,n represents a mortality law of the property then the D(i) vector appears as follows for k dollars placed in service.

Beginning of period

D'(1)=[0,kf(1),kf(2),....,kf(n)]

D'(2)=[kf(1),kf(2),kf(3),....,kf(n),0]

D'(3)=[kf(1)+kf(2),kf(3),....,kf(n),0,0]

End of Period

D(2)=[kf(1),kf(2),kf(3),....,kf(n),0]

D(2)=[kf(1)+kf(2),kf(3),....,kf(n)0,0]

D(3)=[kf(1)+kf(2)+kf(3),....,kf(n),0,0,0]

Let matrix P be dimensioned n x n with ones in the lower left diagonal and a one in the upper left hand corner. (21, p. 716)

$$P = \begin{vmatrix} 1 & 0 & 0 & 0 & \dots & 0 \\ 1 & 0 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 \\ . & . & . & . & \dots & . \\ . & . & . & . & \dots & . \\ 0 & 0 & 0 & 0 & \dots & 0 \end{vmatrix}$$

Multiplying vector D with matrix P yields the following result.

$$D(1)=D'(1)P$$

$$D(2)=D(1)P$$

$$D(3)=D(2)P$$

....

$$D(k)=D'(1)P^k$$

The previously developed expressions may be more clearly understood by considering the following example. Let f(x), the percentage retirements per period, be defined as

x	f(x)
1	.2
2	.3
3	.5

such that if k, the initial placement, is $100 then D'(1)=(0,20,30,50). Let matrix P be constructed as

$$P=\begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{vmatrix}$$

The values of D'(x) and D(x) at the beginning and end of a time period for a vintage account are the following.

Beginning of Period

D'(1)=(0,20,30,50)

D'(2)=(20,30,50,0)

D'(3)=(50,50,0,0)

End of Period

D(1)=(20,30,50,0)

D(2)=(50,50,0,0)

D(3)=(100,0,0,0)

It is observed that

$$D(1)=D'(1)P$$

$$D(2)=D(1)P$$

$$D(3)=D(2)P$$

and by substitution that $D(k)=D'(1)P$.

It is obvious that the stochastic matrix P controls the flow of property through vector D. The concepts, where footnoted, were described in articles by Zannetos (21) (22). His papers suggested that with such a mathematical description, new procedures for calculating depreciation charges as well as new methods for allocation might result. Zannetos also believed the established rigor of discrete mathematics as related to the stochastic matrix would be of help in solving problems associated with the areas of depreciation and life estimation.

In his description of the property vector D, Zannetos made no attempt to differentiate between the beginning or end of one transaction period. Because of this, the two formulas he developed to calculate depreciation charges appear to be in error. The first formula described was to calculate the depreciation charge of a vintage account using the straight line method, average life procedure. Letting DC(x) represent the depreciation charge in year x, the formula was $DC(x)=[D'(1)-D(x)]\,N'/n$ where $N=(0,1,2,\ldots,n)$ and N' is its transpose. The denominator should be the average service life rather that the maximum life n. Let ASL denote average service life, the $ASL = \sum xf(x)$ where $x=1,2,3,...,n$ and the correct formula written as $DC(x) = [D'(1)-D(x)]\,N'/ASL$. His second formula, a calculation of depreciation charge for large x, was $DC(x) = (D(x)-D(x+1))\,N'/n$. In a no growth continuous property account $D(x)$ and $D(x+1)$ are equal for large x and therefore his calculated depreciation charge is always zero. This cannot be correct since for the depreciation reserve to stabilize, which it does for the straight line method, average life procedure, the dollars of retirement equal the dollars of annual depreciation charge. It is also observed that the denominator should be ASL. The correct formula is $DC(x) = [D'(x)-D(x)]\,N'/ASL$. There is no provision for salvage value consideration in either of his formulas, so it was assumed in analyzing the equations that salvage value was zero, and both corrections are based on this assumption.

Deterministic Elements of a Stochastic Matrix for a Continuous Account

The D property vector representing a continuous no growth property account operating with mortality law $f(x)$, $x=1,2,3,...,n$, acts in the following manner.

Beginning of period

$$D'(1)=[0, kf(1), kf(2), kf(3), \ldots, kf(n)]$$

$$D'(2)=[0, kf(2)+kf(1)f(1), kf(3)+kf(1)f(2), kf(4)+kf(1)f(3), \ldots, kf(1)f(n)]$$

$$D'(3)=[0, kf(3)+kf(1)f(2)+\{kf(2)+kf(1)f(1)\}f(1), kf(4)+kf(1)f(3)+\{kf(2)+kf(1)f(1)\}f(2), \ldots, \{kf(2)+kf(1)f(1)\}f(n)]$$

END OF PERIOD

$$D(1)=[kf(1), kf(2), kf(3), kf(4), \ldots, kf(n), 0]$$

$$D(2)=[kf(2)+kf(1)f(1), kf(3)+kf(1)f(2), kf(4)+kf(1)f(3), \ldots, kf(1)f(n), 0]$$

$$D(3)=[kf(3)+kf(1)f(2)+\{kf(2)+kf(1)f(1)\}f(2), \ldots \{kf(2)+kf(1)f(1)\}f(n), 0]$$

The $d(r)$ position now may represent the property to be renewed. At the beginning of the period this value is always zero. Whereas, at the end of the period some property may have retired and $d(r)$ represents this amount of retirement or the amount to be renewed. This amount is placed in service and distributed according to its mortality law as illustrated in the previous equations.

If a matrix P is defined as

$$P=\begin{vmatrix} f(1) & f(2) & f(3) & \ldots & f(n) & 0 \\ 1 & 0 & 0 & \ldots & 0 & 0 \\ 0 & 1 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 1 & \ldots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ 0 & 0 & 0 & \ldots & 1 & 0 \end{vmatrix}$$

then

$$D(1)=D'(1)P$$

$$D(2)=D(1)P$$

$$D(3)=D(2)P$$

. . . .

$$D(k)=D'(1)P^k$$

In order to more closely follow the aforementioned development consider the following example. Let f(x), the mortality distribution, be defined as

x	f(x)
1	.6
2	.4

such that if k, the amount of original placement, is $100 the D'(1)=(0,60,40). Let matrix P be constructed as

$$P=\begin{vmatrix} 0.6 & 0.4 & 0.0 \\ 1.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 \end{vmatrix}$$

The values of D'(x) and D(x) at the beginning and end of a time period respectively for a no growth continuous account are

Beginning of Period

D(1)=(0,60,40)

D(2)=(0,76,24)

D'(3)=(0,69.6,30.4)

End of Period

$D(1)=(60,40,0)$

$D(2)=(76,24,0)$

$D(3)=(69.6,30.4,0)$

As was noted in the previous vintage account

$$D(1)=D'(1)P$$

$$D(2)=D(1)P$$

$$D(3)=D(2)P$$

and by substitution that $D(k)=D'(1)P^k$.

A matrix P, nearly identical, was described by Ijiri (10) when he analyzed the pattern of periodic reinvestments in depreciable assets when the amount of reinvestment was set equal to the depreciation charge. The difference in the two matrices was that the first row of the Ijiri matrix represented the portion of property to be depreciated in the x year, rather than the property physically retired. An identical matrix P was described in Feller as a recurrent set of events and residual waiting times (7, p. 381).

Approximations

The discussion thus far has been a description of the basis of another concept of property retirement simulation. During construction of the property account generator it was recognized that some adjustments must be made in order to conform to common practice. Such changes were also required if the results obtained were such that they could be compared to the few cases which had been previously calculated. Accordingly, this section discusses the required approximations, their conceptual development, and how they differ with the state of the art as observed today.

The stochastic matrix $P'=P$ for a continuous account when K approaches infinity may be described as

$$P' = \begin{vmatrix} Q(1) & Q(2) & \cdots & Q(n) \\ Q(1) & Q(2) & \cdots & Q(n) \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ Q(1) & Q(2) & \cdots & Q(n) \end{vmatrix}$$

It has been well established and documented by Cox (6), Feller (7), and Kimball (11), that $Q(1)=1/\sum xf(x)$ $x=1,2,\ldots,n$. $Q(1)$ is commonly called the retirement rate and is equal to the inverse of the average service life of the property.

In considering a vintage account as controlled by matrix P, it is implicit that all retirements occur at the end of each interval. This creates a stair-stepped survivor curve as shown in Figure 1. The retirement rate equals $1/\sum xf(x)$ $x=1,2,\ldots,n$.

Common practice and calculation of the retirement rate are based on the survivor curve shown in Figure 2.

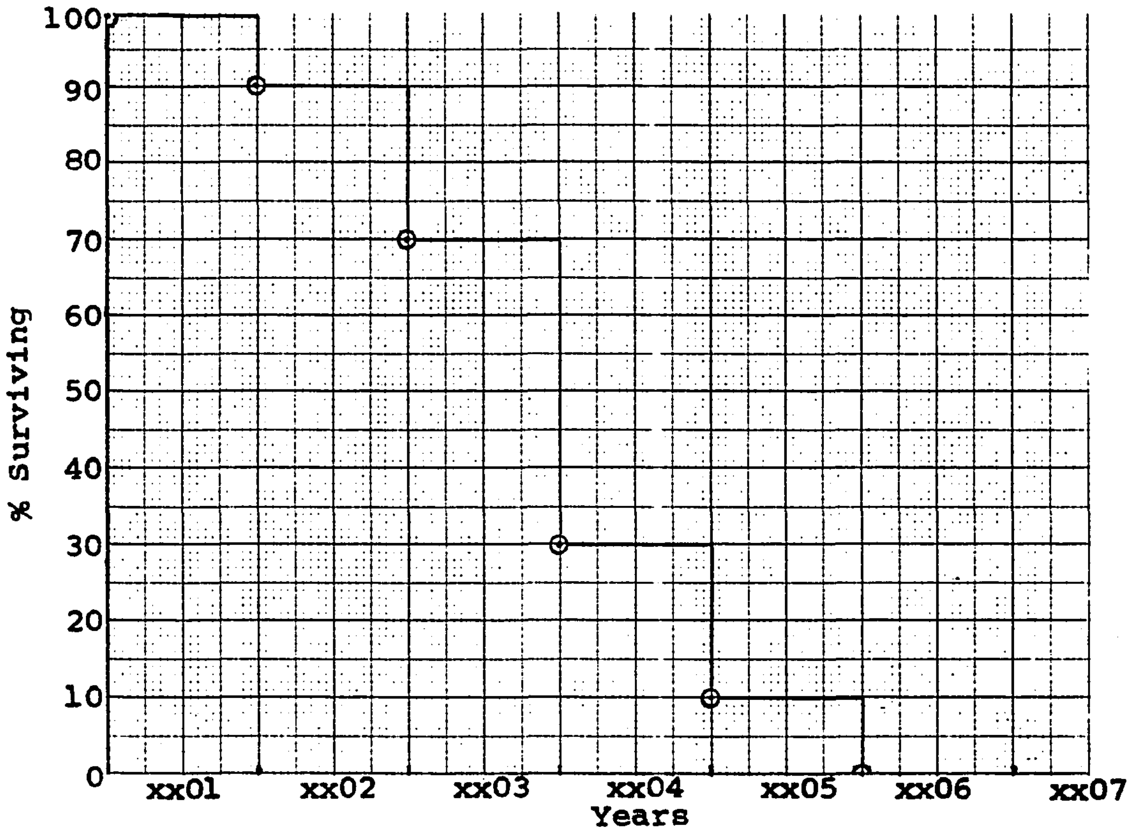

Figure 1. Stair Stepped Survivor Curve

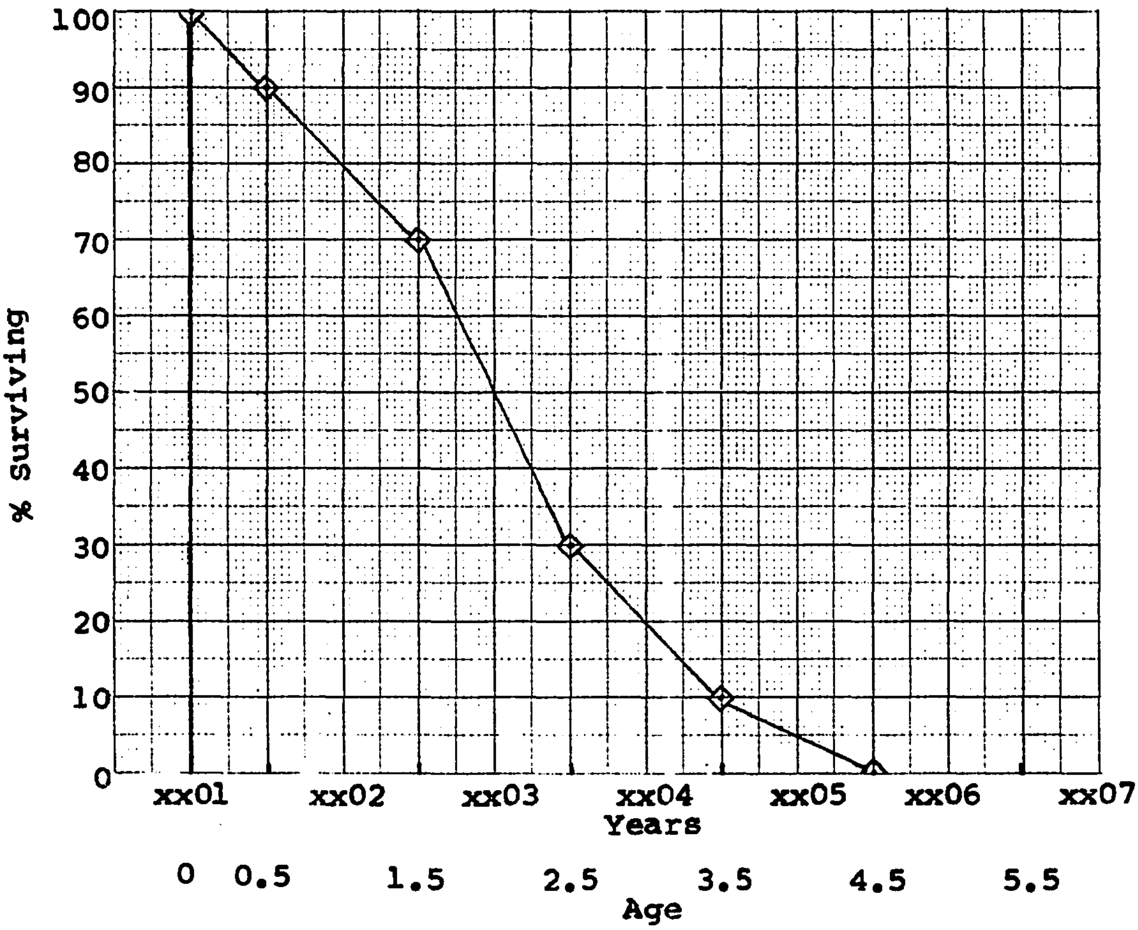

Figure 2. Regular Survivor Curve

The $ASL=(f(1)/4)+\sum(x-1)f(x) \quad x=1,2,...,n$. It is assumed that the original placement occurs in the middle of the first interval, that retirements are uniform over an interval, and that the end of an interval or time period may also be represented as the age of the property. The retirement rates of these two survivor curves differ quite markedly. Their denominators which are the respective average service lives, have a difference of $1-f(1)/4$. Appendix II.A. illustrates these calculations. This quite large difference in retirement rates is all due to the aforementioned assumptions.

In order to simulate a continuous property account, an assumption is required relative to the timing of replacements. It is a reasonable proposition that retirements occur uniformly during an age interval, and in order to keep the property accounts at full service, any retirement must be immediately replaced.

If $kf(1)$ retire during the first interval, then an amount of $kf(1)$ must be immediately replaced. If there are retirements from these renewals, then an additional $kf(1)^2$ must also be replaced. Continuing this argument adinfinitum, a sum of a geometric series is obtained whose value approaches $kf(1)/(1-f(1))$.

For the first age interval of any vintage account, whether it is a replacement or initial property placement, it will be assumed that an additional amount equal to the expected retirement will be placed with the original placement. Thus, all the property in the account at the end of the first interval will have an age of one half. This assumption now changes the first row of the P matrix to

$g(x)=f(x+1)/(1-f(1)) \quad x=1,2,\ldots,n-1.$

$$P'=\begin{vmatrix} g(1) & g(2) & g(3) & \cdots & g(n-1) & 0 & 0 \\ 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 & 0 \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdots & 0 & 1 & 0 \end{vmatrix}$$

The Q' matrix of P' defines $Q'(1)=1/\sum xg(x)$ where $x=1,2,\ldots,n-1$. The difference between the average service life as calculated in common practice and the inverse of $Q'(1)$ plus the adjustment is barely perceptable. Appendix II.B. demonstrates the aforementioned calculations. If $f(1)$ equals zero there is no difference in the two average service life calculations.

Stochastic Elements of a Stochastic Matrix

From previous discussion it has already been established that the function of matrix P is to control the flow of property through vector D. The use of ones in the lower left diagonal make the system deterministic since after the lapse of a period the amount of property with x years of life remaining will automatically have only x-1 years of service left.

Changing the matrix P, yet still retaining its stochastic properties, as shown below, creates a system where the property in vector D would not retire as fast as would be expected. Such a system has been previously described as non deterministic since it doesn't automatically shift the property down the D vector as before.

$$P = \begin{vmatrix} g(1) & g(2) & g(3) & \cdots & 0 & 0 \\ p & q & 0 & \cdots & 0 & 0 \\ 0 & p & q & \cdots & 0 & 0 \\ \cdot & \cdot & p & \cdots & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ 0 & 0 & 0 & \cdots & p & q \end{vmatrix}$$

The P matrix could also be changed to

$$P = \begin{vmatrix} g(1) & g(2) & g(3) & g(4) & \cdots & g(n-1) & 0 & 0 \\ 1 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ q & p & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & q & p & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & q & p & \cdots & 0 & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdots & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdots & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdots & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & \cdots & q & p & 0 \end{vmatrix}$$

which would speed up the retirement of property in vector D.

The selection of q only to the immediate left or right of the lower left diagonal reflects the viewpoints that during any time period the remaining life of part of the property will gain or lose one period. This change of the element of P yields a powerful tool to model the effect of any force which either enhances or delays property retirement.

Features and Capabilities of the Property Account Generator

The basis of the property retirement simulator is the control of the D vector by the P matrix. The general form of the P matrix which was selected is the following.

$$P = \begin{vmatrix} g(1) & g(2) & g(3) & g(4) & \cdots & g(n-1) & 0 & 0 \\ b & c & 0 & 0 & \cdots & 0 & 0 & 0 \\ a & b & c & 0 & \cdots & 0 & 0 & 0 \\ 0 & a & b & c & \cdots & 0 & 0 & 0 \\ \cdot & & & & & & & \cdot \\ \cdot & & & & & & & \cdot \\ \cdot & 0 & 0 & 0 & \cdots & a & b & c \end{vmatrix}$$

The computer program was developed such that the elements within the P matrix will model forces which delay or enhance property retirements. This section describes the various features of the property account generator.

There are several features of the property account generator which are not related directly to the P matrix. The first feature is the capability to specify one characteristic of the property account; the growth pattern. This pattern may be changed any number of times in any time period. The property account was pre-determined to be open ended. A depreciation reserve account was constructed and was pre-determined to use the straight line method, average life procedure of depreciation. The depreciation rate could be changed in any period any number of times. Another set of features is that the property accounts and the reserve accounts for each year are kept separate from one another. This provision is mandatory if evaluation of subjects within the life estimation area and observation of the depreciation reserve characteristics are to be accomplished. Other features revolve around the concept that for the property account simulator to be useful it must be able to generate many sets of property records given a set of specifications for the property account and mortality characteristics. Thus, the property account generator was constructed with these characteristics. The property account generator also has routines which gather designated reserve and renewal information. It then organizes these facts into tables and histograms.

The remaining features built into the property account generator revolve around construction of the elements of matrix P. Recognizing that even with no major forces working to enhance or delay retirements, there are minor fluctuations in the system which have a small effect on the flow of property through vector D. A provision has been made to specify a distribution from which a periodic random selection is made in order to simulate these minor fluctuations in property retirements.

Major variations in property retirements, previously referred to as trends, may be incorporated into the P matrix. Provisions were made to assign trends in any designated period and also to be able to change trend percentages in any desired period.

The last two features of the retirement experience generator relate to the mortality law. The group of mortality laws utilized are called the Iowa Survivor Curves (18, p. 70,72). These curves are described as right, left and symmetric modal by R, L, and S respectively. A subscript follows to designate the shape of the distribution. Lastly, the average service life is added to the information about the mortality law. An S(3)-5 would describe a symmetric mortality law of shape three and average service life of five years. Provision has been made to change not only the type and shape of the distribution, but also the average service life. This feature together or singularily may be utilized in any designated time period as frequently as desired. If a change in average service life was required, then previous account's flow through rates were adjusted to the difference of the inverse of the old average service life and the new average service life. The calculations are shown in Appendix II.C.

With the aforementioned capabilities it was expected that a property account would be developed whose characteristics followed actual accounts closely enough that tests could be conducted on the simulated accounts, and meaningful conclusions could be drawn. A detailed description of the computer program is found in Appendix I.

RESULTS AND DISCUSSION

The results of the five simulations trials are presented in this section. The major purpose of these trials was to evaluate the effectiveness of the retirement experience generator. It was also desired to use these trials to make a cursory inspection of the reserve behavior. The format used to describe these five trials will be a general description of the trial or trials, a graphical presentation of the results, and a discussion of the significant information illustrated. The final section of this chapter presents suggestions for improvement of the property account generator.

Before the five major trials were run, a preliminary set of trials was utilized to provide a better understanding of the computer program. It was observed that one simulation or property account construction of length twenty years cost seventy cents. The cost of a longer time period increased exponentially. It was decided that any test to be run would be accomplished using a sample size of thirty and a maximum of twenty years of property account generation. A larger sample of size fifty was tried, but it seemed to provide very little extra information considering the extra cost involved. Because the property account generator was limited to twenty years, the trials were run using a mortality dispersion with an average service life of five years unless otherwise specifically stated. The average service life of five years enabled the account to cover four life cycles of the property. From previous experience this seemed reasonably adequate for observation purposes when studying the behavior of the depreciation reserve.

These preliminary runs were also used to select a probability distribution which represented the random fluctuations of property flowthrough to retirement. The four distributions tested are listed below. The percentage fluctuation is represented by x and Pr(x) denotes the probability of its occurrence.

1

x%	Pr(x)
-20	.025
-10	.10
-5	.25
0	.25
5	.25
10	.10
20	.025

2

x%	Pr(x)
-20	.05
-10	.10
-5	.20
0	.30
5	.20
10	.10
20	.05

3

x%	Pr(x)
-10	.10
-5	.25
0	.30
5	.25
10	.10

4

x%	Pr(x)
-5	.05
-3	.20
0	.50
3	.20
5	.05

The fourth distribution, which hereafter will be referred to as distribution four, appeared to yield what was thought to be a reasonable approximation of the random fluctuations.

Before any critique may be made on the correctness of selecting distribution four, it must be observed that the choice of elements has a direct bearing on the result of such a selection. Note carefully that the P matrix was selected to be of the following form.

$$P=\begin{vmatrix} g(1) & g(2) & g(3) & g(4) & \cdots & g(n-1) & 0 & 0 \\ b & c & 0 & 0 & \cdots & 0 & 0 & 0 \\ a & b & c & 0 & \cdots & 0 & 0 & 0 \\ 0 & a & b & c & \cdots & 0 & 0 & 0 \\ \cdot & & & & & & & \cdot \\ \cdot & & & & & & & \cdot \\ \cdot & 0 & 0 & 0 & \cdots & a & b & c \end{vmatrix}$$

This form of the retirement model implies the effect of property flowthrough to retirement is felt equally in each year. It is recognized that this is one of many ways in which the matrix could be built. A possible suggestion would be to establish the b values to be exponentially dampened. For example, $b(r)=1-[1-b(2)]\exp(1-r/2)k$ where $b(r)$ is the b value in row r, and k is some dampening constant. This would have the effect of retirements in later years not as dependent on present flowthrough characteristics. Obviously there are many positions not utilized in the present construction of the P matrix which implies there are a multitude of unexplored possibilities to model flowthrough of property retirements. The simulation and mathematical analysis of such possibilities would result in a better retirement model. After some deliberation, it was believed the aforementioned P matrix would be a reasonable retirement model, and that distribution four would adequately represent minor fluctuations in property flowthrough.

The reserve account specifications were pre-determined to be open ended using the straight line method, average life procedure of depreciation. The property account growth was zero for all trials except trial three. The basic mortality law was an Iowa Survivor Curve S(3) with varied parameters of average service life and major trends. The remaining specifications are listed in the discussion or the figures which illustrate the results.

Trial one illustrated the effect varying the average service life had on the behavior of the depreciation reserve. The data which has been listed in Appendix IV.A. is graphically displayed in Figure 3.

The second trial ascertained the effect on depreciation reserve by enhancing the property flowthrough by three, five, and seven percent after year ten. This might be analogous to the effect of unexpected technological obsolescence. The depreciation rate was kept constant. The graphical results are shown in Figure 4, and the numerical data is listed in Appendix IV.B.

Trial three demonstrated the effect of property account growth on the depreciation reserve. The numerical data which is found in Appendix IV.C. is displayed in Figure 5.

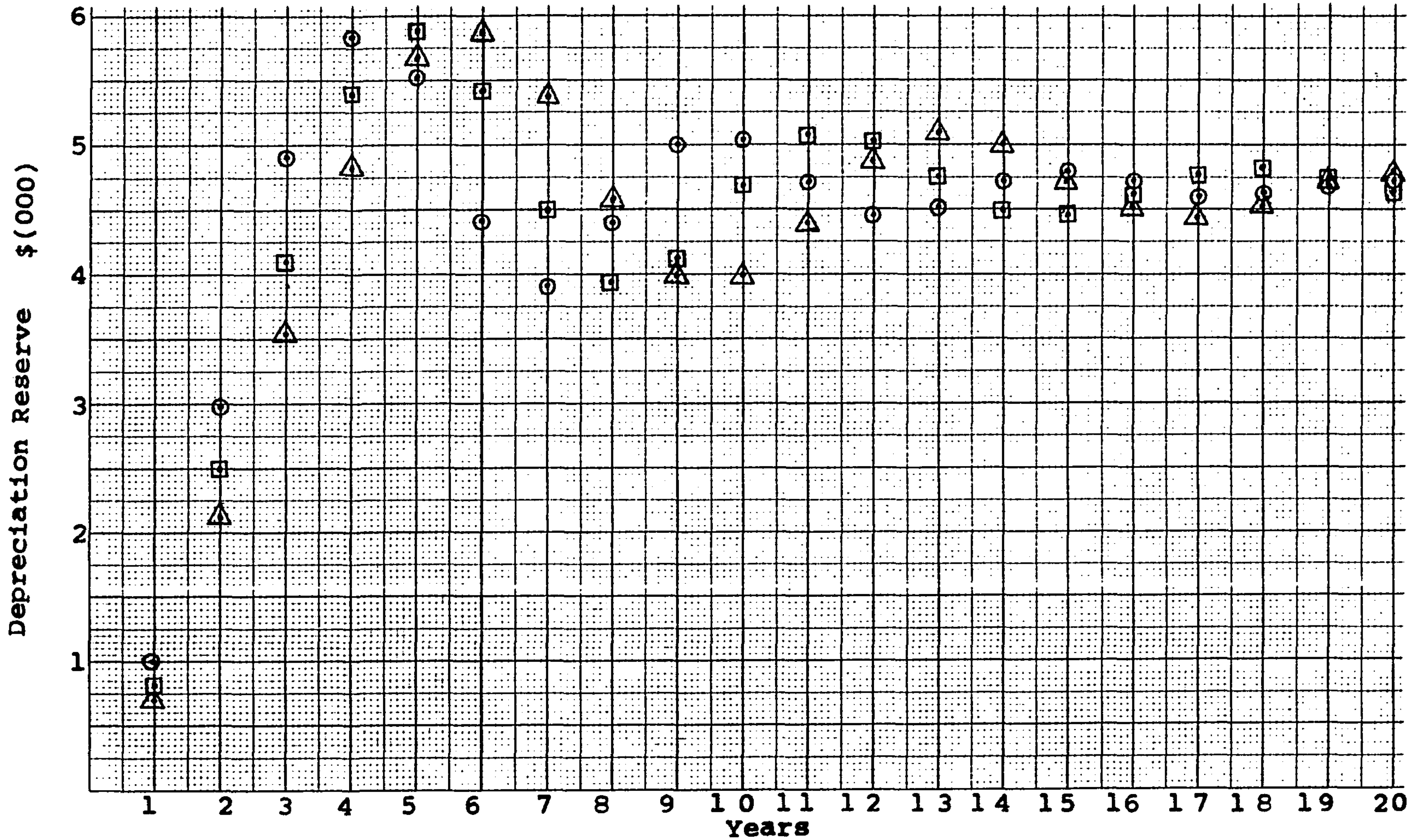

Figure 3. Depreciation Reserve

Initial Placement: $10,000

Mortality Characteristics:
⊙ =S(3)-5
⊡ =S(3)-6
△ =S(3)-7

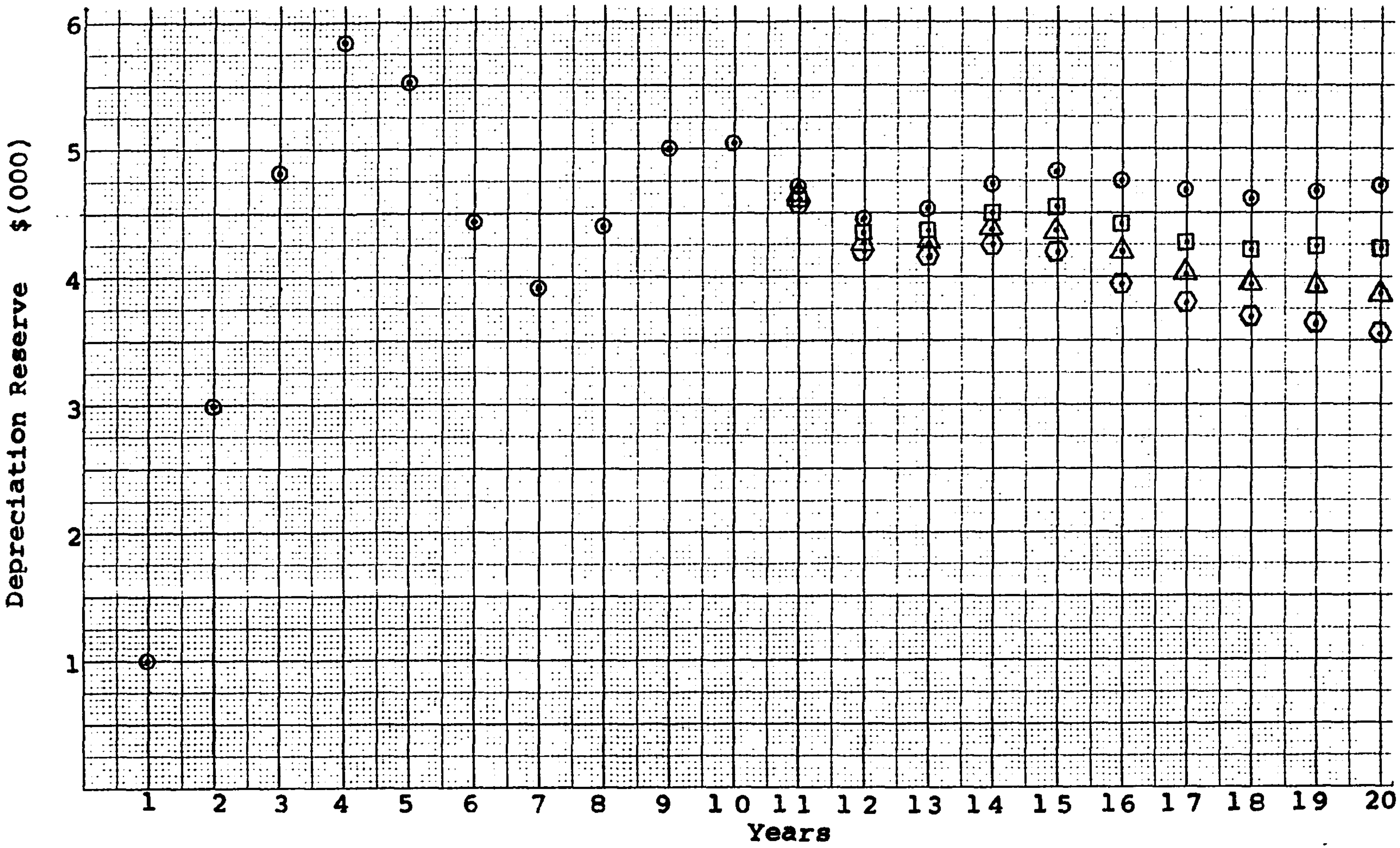

Figure 4. Depreciation Reserve
Initial Placement: $10,000

Mortality Characteristics:
S(3)-5

Trends:
⊙ =0% △ =5%
⊡ =3% ⬡ =7%

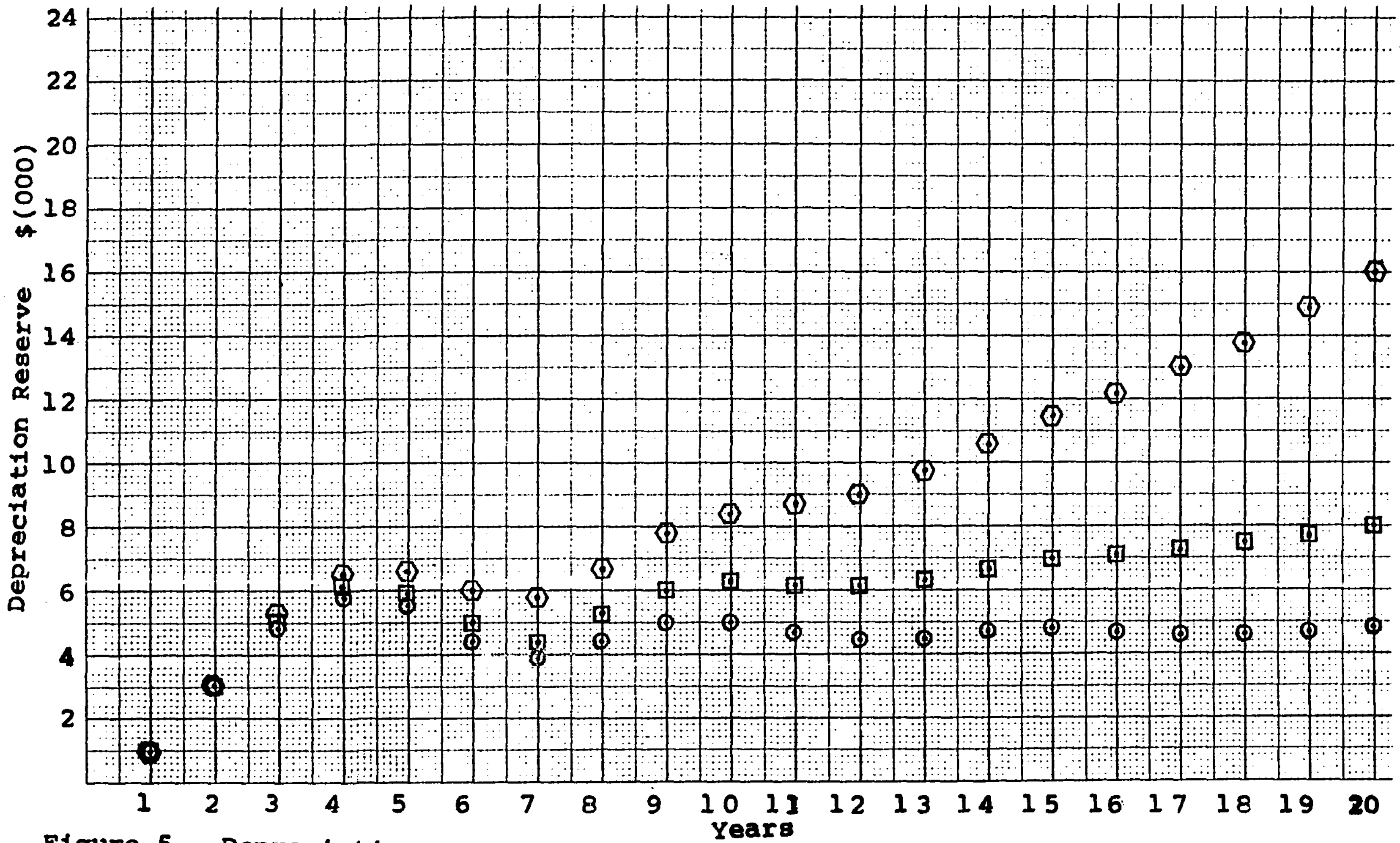

Figure 5. Depreciation Reserve
Initial Placement: $10,000

Mortality Characteristic: S(3)-5

Account Growth: ⊙ =0%
⊡ =3%
⬡ =7%

The first three trials were necessary in order to establish confidence in the accuracy of the property account generator. The results of trial one were compared to Winfrey's (19, p. 49) work. Winfrey had established the steady state depreciation reserve factors for the S(3) Iowa Survivor Curve to be $4,702 for an initial placement of $10,000. The results of trial one compared favorably to this value.

The second trial demonstrated the use of the non-deterministic major trends of property mortality characteristics which would either enhance or delay property flowthrough to retirement. These trends have the same effect as changing the average service life of the mortality characteristics of the property account. The advantage of this feature is that it can generate property accounts in which the sensitivity of life analysis techniques may be tested.

Here it is important to note that when a change of mortality characteristic takes place, by proper manipulation of the elements of the P matrix, all property previously placed in service can be affected by this change. This is a very powerful feature of the stochastic matrix, controlling the property flowthrough to retirement. This particular capability is non-existent in previously constructed property retirement simulators which utilize the Monte Carlo technique to create minor fluctuations in property retirements.

Since major trends may be sinusoid in nature, it is suggested that such a feature be incorporated in the program. As the generator is constructed presently, sinusoid trends may be simulated, but at the expense of excessive preparation of input information. The third trial merely demonstrated the fact that the growth factor feature of the property account generator was working properly.

The fourth trial demonstrated the behavior of the depreciation reserve using varying conditions of mortality characteristics and depreciation rate specifications. The mortality characteristics are changed in the tenth year the account was placed in service. The three property account specifications all are relative to this year of change. The first specification was that the depreciation rate would remain unchanged. The second specification was that the depreciation rate would correctly change in the same year the mortality characteristic changed. The final specification was that the depreciation rate would correctly change, but would be delayed two years from the change of the mortality characteristic. Figure 6 exhibits the depreciation reserve behavior under these conditions. The data is listed in Appendix IV.D.

The fourth trial is the first of many expected deterministic simulations which attempt to find answers to the question of what happens to the depreciation reserve under a va-

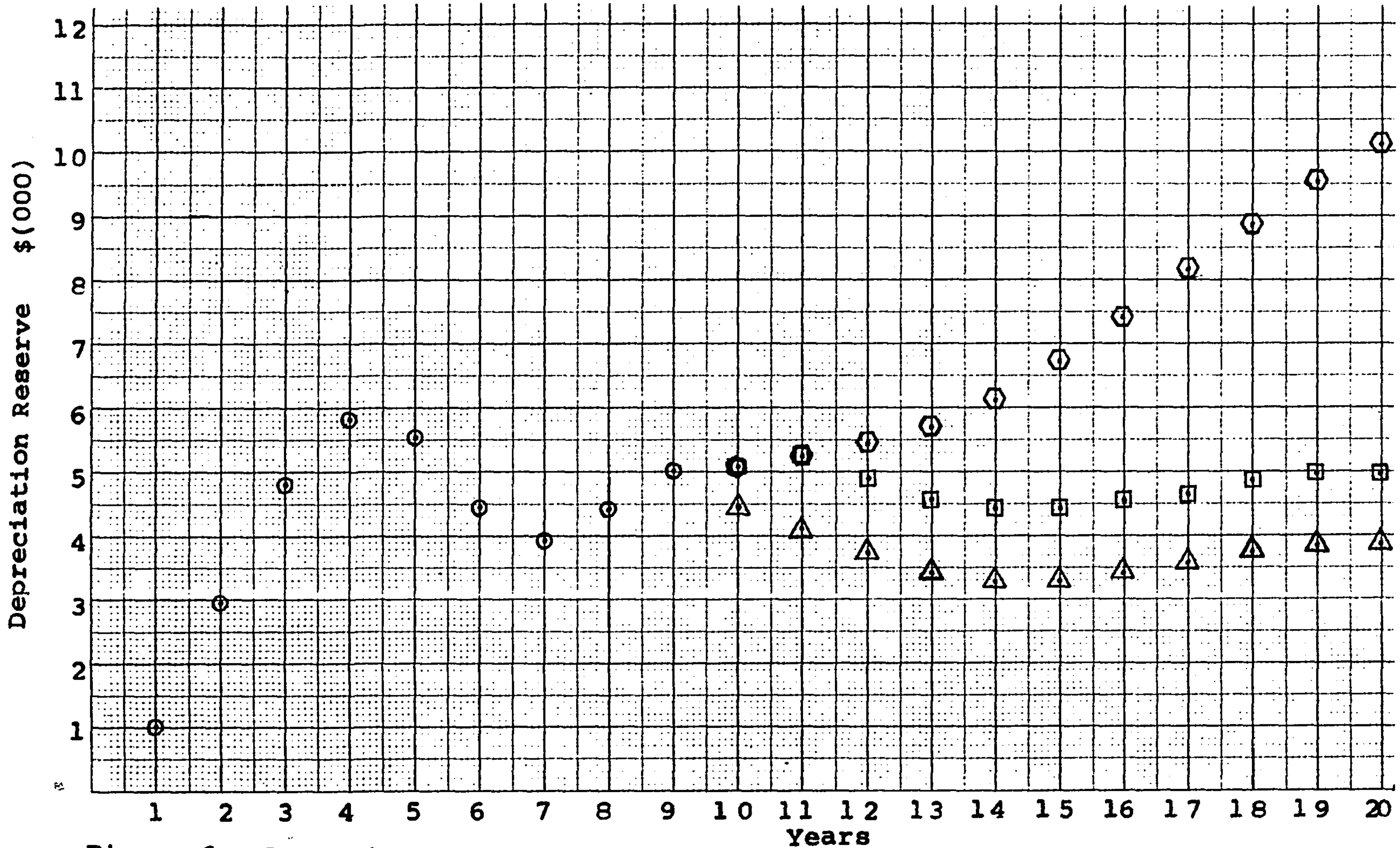

Figure 6. Depreciation Reserve
Initial Placement: $10,000
Mortality Characteristics:
S(3)-5: Years 1-9
R(1)-7: Years 10-20

Depreciation Rate:
⊙ =.2: Years 1-20
△ =.2: Years 1-9
.143: Years 10-20
⊡ =.2: Years 1-11
.143: Years 12-20

riety of complex situations. Even in this trial, it was observed that the depreciation reserve is not self correcting if the correction is delayed when the straight line average life procedure of depreciation is used. How other depreciation methods and procedures affect the depreciation reserve under a set of complex economic conditions is certainly a question which now has a better chance of being answered using the presently constructed property account generator. A specific example of one such question would be; is the remaining life procedure really a self correcting technique as it is presently being applied?

The fifth trial used distribution four to generate minor fluctuations in property flowthrough to retirement. Samples of size thirty of the depreciation reserve were taken for years eleven through twenty from each of four mortality characteristics. The property account specification were held constant. Information from an S(3)-5 was collected. Using this mortality dispersion as a base, after the tenth year, the property flowthrough was enhanced by trends of three, five, and seven percent. The information from these three runs was also collected and recorded. Appendix VI summarizes the means and variances of the depreciation reserve distribution as well as listing the pure values of the depreciation reserve.

Recognizing that this is the first time depreciation reserve distributions have been available for analysis, there were questions concerning the type of distribution they were, the equality of their variances, and the significant difference of the means of the respective distributions.

It was hypothesized that the depreciation reserve distributions were normal. Utilizing a chi-squared goodness-of-fit test described by Ostle (15, p.126), it was concluded that such a hypothesis was reasonable. The results of the test are listed below.

Hypothesis(+): Depreciation Reserve Distributions are Normal

Mortality Characteristic: S(3)-5

significance level=0.01

Trends Years	0%	3%	5%	7%
11	+	+	+	+
15	+	+	-	+
20	+	+	-	-

significance level=0.05

Trends Years	0%	3%	5%	7%
11	+	+	+	+
15	+	+	-	+
20	-	+	-	-

significance level=0.10

Trends / Years	0%	3%	5%	7%
11	+	+	+	+
15	+	+	-	+
20	-	-	-	-

Observing the aforementioned table again, it seemed that for the assumptions of normality to hold, as the account time progressed or as the trend percentage increases, the significance level must remain quite low. What model features were causing this, and why, was not immediately apparent. One conjecture is that the longer the trend is in effect, the more pronounced the skewness of the depreciation reserve distribution.

A second hypothesis stated was that the variances of the depreciation reserve distribution each year for the zero, three, five, and seven percent trends were equal. Bartlett's test (15, p. 136) confirmed this hypothesis for years eleven, fifteen, and twenty at significance levels of 0.01,0.05, and 0.10.

It was hypothesised that it would take several years before the magnitude of the depreciation reserve for the three percent trend would become statistically significant from the zero percent trend. Let $r(y\%, z)$ represent the pure depreciation reserve calculation for a $y\%$ trend at age z. Let $\bar{x}(y\%, z)$ represent the mean of the sample of the depreciation reserve for a $y\%$ trend at age z. The first hypothesis was that $\bar{x}(3\%, z) < r(0\%, z)$. Using a t-test as described by Ostle (15, p. 133), the following results were calculated using the data from Appendix VI and listed below.

Hypothesis(+): $r(0\%,z) > \bar{x}(3\%,z)$ for a S(3)-5

Significance level \ Year	11	12	13 thru 20
0.01	-	-	+
0.05	-	+	+
0.10	-	+	+

The test exhibited significance at all levels quite quickly as time progressed. $\bar{x}(5\%, z)$ and $\bar{x}(7\%, z)$ were not tested since they would show significance even more quickly. It is conjectured that for property accounts with longer average service life, a significant difference in means may not be reached as quickly in terms of years. In terms of percent of average service life, it is expected that the analysis would be nearly identical.

There was a casual observation that the $\bar{x}(k\%, z)$ was greater than $r(k\%, z)$. This seemed to violate a previously stated intuition that the central tendency of the depreciation reserve would be near the pure depreciation value. A t-test was conducted to ascertain if $\bar{x}(k\%,z) > r(k\%,z)$. It was determined that such was the case at a significance level of 0.001.

This unique situation is not surprising if the P matrix is closely observed.

$$P = \begin{vmatrix} g(1) & g(2) & g(3) & g(4) & \cdots & g(n-1) & 0 & 0 \\ b & c & 0 & 0 & \cdots & 0 & 0 & 0 \\ a & b & c & 0 & \cdots & 0 & 0 & 0 \\ 0 & a & b & c & \cdots & 0 & 0 & 0 \\ \cdot & & & & & & & \cdot \\ \cdot & & & & & & & \cdot \\ \cdot & 0 & 0 & 0 & \cdots & a & b & c \end{vmatrix}$$

Using the fourth distribution to describe the fluctuations in property flowthrough, it is observed that in row two, b will be the value 1, seventy-five percent of the time; the value 0.97, twenty percent of the time; and the value 0.95, five percent of the time. The average value of b in the second row is less than one. This implies a delay of property flowthrough to retirement. An examination of rows beyond the second row finds the average value of b equal to one. The implication is that there is no delay or enhancement of property flowthrough. With a slight delay in property retirement due to row two of the P matrix, the depreciation reserve in non-deterministic simulations would be higher than expected pure depreciation reserve.

The implications of the previous discussion are quite startling. They suggest that the previous levels of depreciation reserve must be corrected upward only because of normal minor fluctuations in the property flowthrough to retirement. It would be quite interesting to observe what impact various minor fluctuation's distributions have on increasing the depreciation reserve level. The mortality characteristics of the property may also be a factor in determining the proper correction factor to apply. Of course, the above two conditions are dependent on the arrangement and calculations of the elements in the P matrix which form the basic retirement model. Though the aforementioned concepts are not results of an in depth study, there seems to be enough evidence to warrant more research in this area because of the impact it may have on governmental and corporate decisions.

Whether a depreciation reserve observation is significantly different or not from some expected level, it still doesn't answer the question of from which set of mortality characteristics such an observation could have come. In an attempt to shed more light in this area the probabilities of misclassification are calculated from the information supplied by Appendix VI.

The probabilities of misclassification are nothing more than determining the percentage chance that an observation was assigned to set A when in reality it belongs to set B. Let $p(0\% | 3\%, x)$ represent the probability of classifying a property account with the 0% trend, given it was the 3% trend which caused that depreciation reserve value. The value x represents the criteria for classification. In this case if the depreciation reserve observed is greater than x, the observation is said to have come from the mortality characteristic with a 0% trend. Conversely, if the depreciation reserve observed is less than x, the observation is placed with the mortality characteristics with a 3% trend. It might be more proper to represent the probability of misclassification as P(0%|3%,x,y) where y denotes the time that the 3% trends has been in effect. Since this portion of the chapter is not an in-depth study of the misclassification problem, but only a small illustration of the type of information which may be derived from interpetation of the output supplied by the property account generator, the former P(0%|3%,x) will be considered sufficient. From Anderson (2, Chapter 6) the mathematics of the probability of misclassification are developed assuming a normal distribution with equal variances. The basic structure of this development as applied to the problem at hand is found in Appendix III. The numerical results of the probabilities of misclassification are listed in Appendix VII, and graphically displayed in Figures 7,8,9, and 10. The value x is in terms of standard deviation from $\bar{x}$(0%, z) for all except Figure 10, where x represents the midpoint between the two means.

The graphs illustrate several interesting concepts. One such concept exhibited is that the closer the criteria is to the mean, the lower the probability of misclassification as the years progress. The converse is also noted that the further away the classification criteria is from the mean,

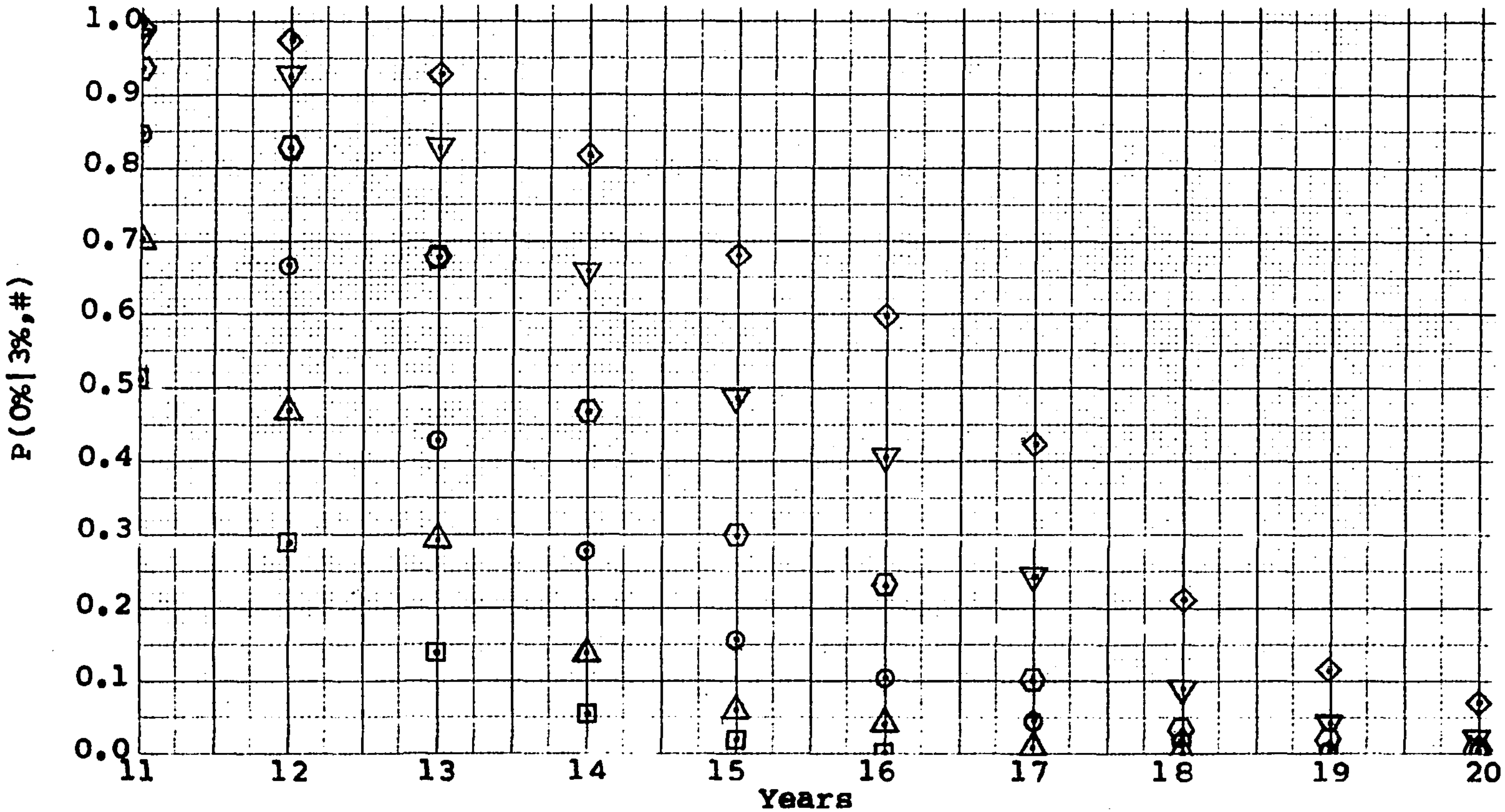

Figure 7. Probability of Misclassification
Initial Placement: $10,000

Mortality Characteristics:
S(3)-5
3% trend 11-20 years

Criteria: # of Standard Deviations from x(0%,z).
⊡ =0.5 ⊙ =1.5 ▽ =2.5
△ =1.0 ⬡ =2.0 ◇ =3.0

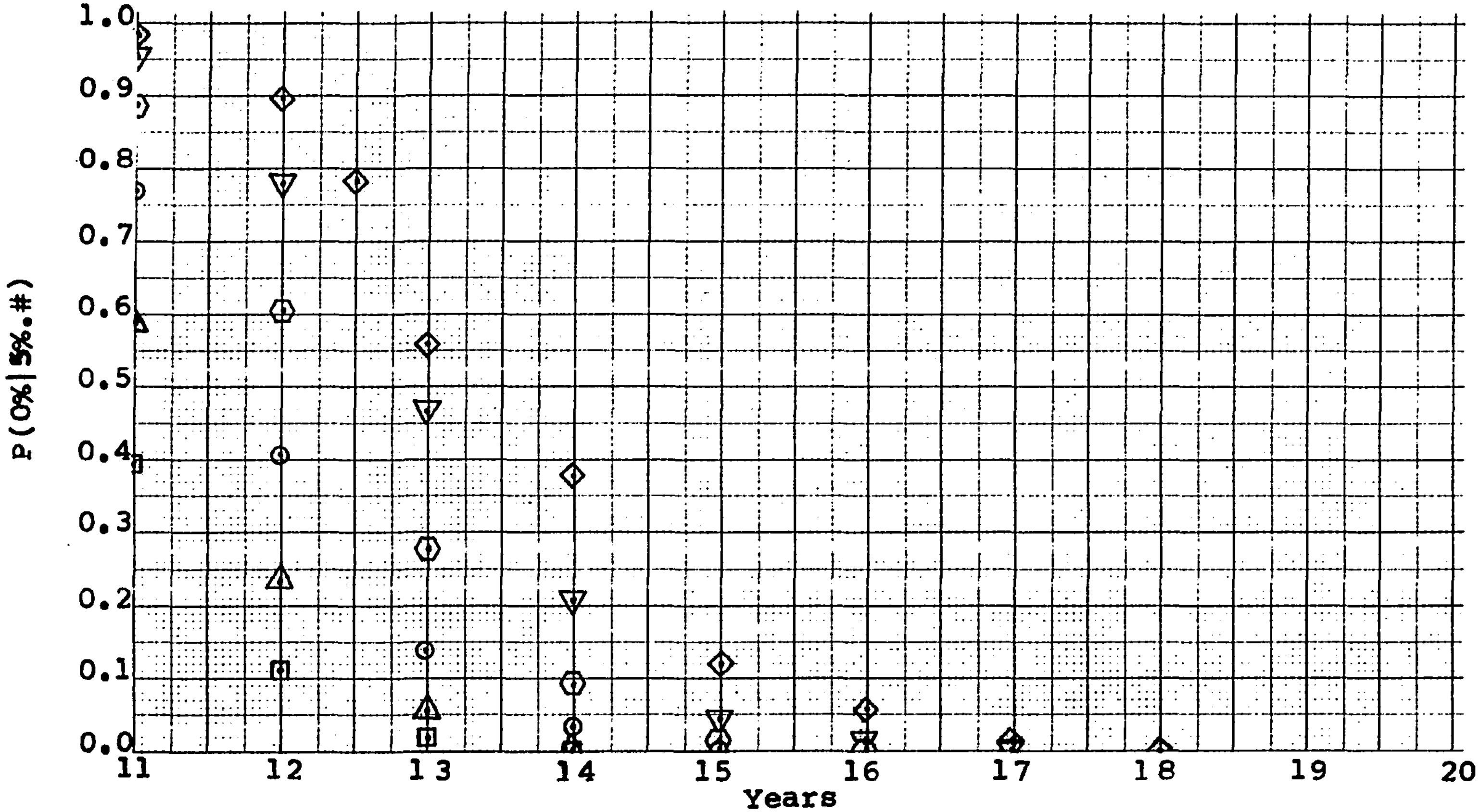

Figure 8. Probability of Misclassification
Initial Placement: $10,000
Mortality Characteristic:
S(3)-5
5% trend 11-20 years

Criteria: # of Standard Deviations from x(0%,z).
⊡=0.5 ⊙=1.5 ▽=2.5
△=1.0 ⬡=2.0 ◇=3.0

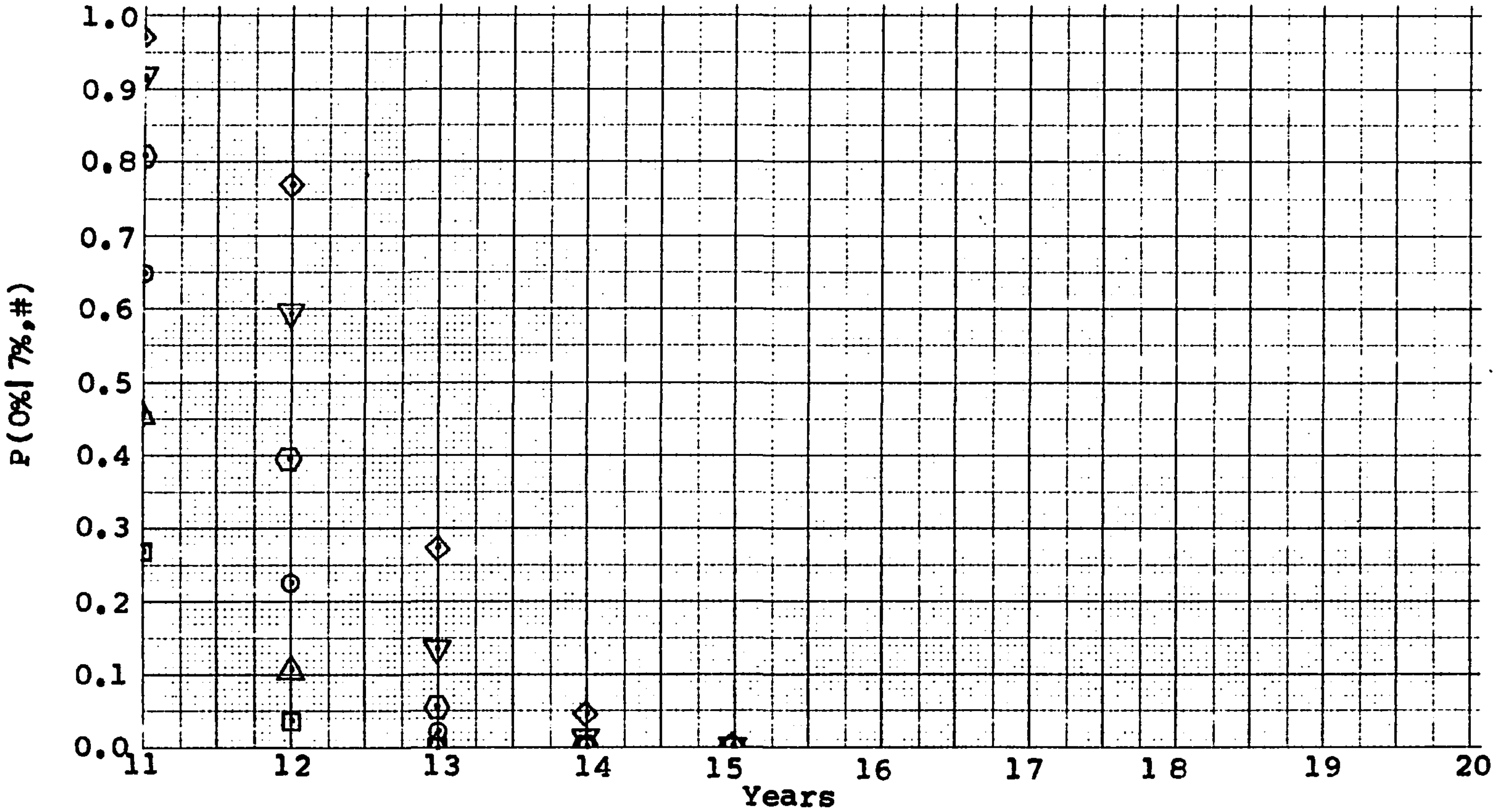

Figure 9. Probability of Misclassification

Initial Placement: $10,000

Mortality Characteristics:
S(3)-5
7% trend 11-20 years

Criteria: # of Standard Deviations from x(0%,z).
⊡=0.5 ⊙=1.5 ▽=2.5
△=1.0 ⬡=2.0 ◇=3.0

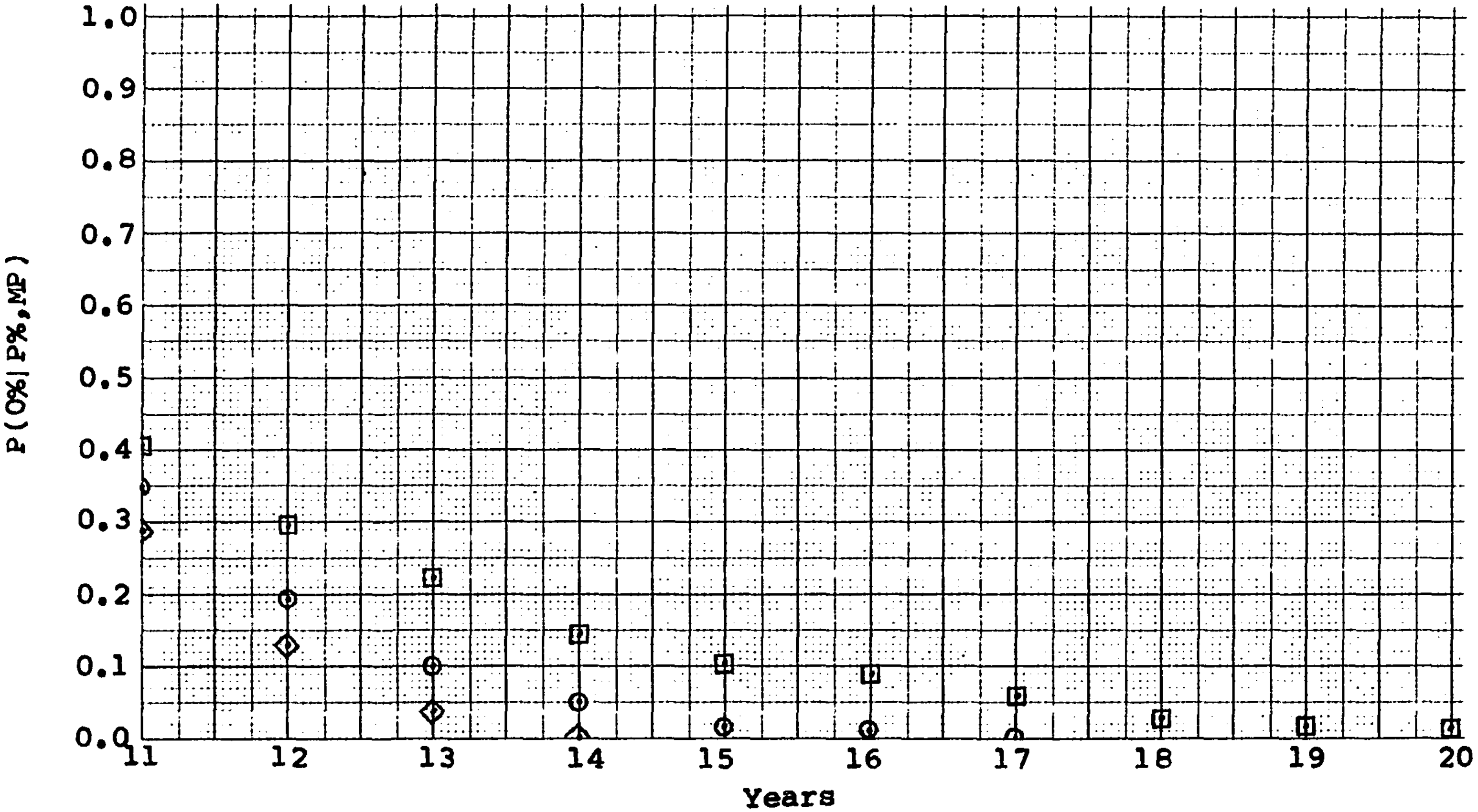

Figure 10. Probability of Misclassification
Initial Placement: $10,000
Mortality Characteristics:
S(3)-5
Trends of P% 11-20 years

Criteria: Mid-point between
x(0%,z) and x(P%,z)
□ -P%=3% trend
⊙ -P%=5% trend
◇ -P%=7% trend

the higher the probability of misclassification. The graphs also exhibit that as the trends become more pronounced, the probability of misclassification is reduced. The information projected also gives a guideline to the degree of sensitivity that may be expected from life analysis techniques which detect changes in mortality characteristics. It is also observed that minor changes in mortality characteristics require a much longer time to become apparent than major changes in mortality characteristics. Yet, the detection of major changes of mortality law do take several years before they even become apparent.

The ideas illustrated by the previous discussion have been intuitively obvious to practitioners in the field for many years. It is significant that this is the first time that actual numbers have been associated with these concepts. The aforementioned discussion was based on mortality characteristics which had a five year average service life. What happens to the probabilities of misclassification when property accounts have larger average service lives may only be conjectured at this point. The speculation is that if the horizontal scale remains in years, that the slopes of the respective lines will be less. On the other hand if the horizontal scale is calculated in terms of percent of average service life, the graphs would be expected to remain the same. Depending on how low a practitioner in the field wished to keep his probability of misclassification, it appears that a significant portion of the life cycle must pass before, at which level a reasonably low probability of misclassification is arrived.

The subject of the probability of misclassification would not be complete unless several other ideas were mentioned. The previous probabilities were calculated for a single observation which was expected to fit in one of two categories. It would seem reasonable that other probabilities could be calculated for a single observation which might be placed in one of several categories. Considering that the information is available about previous depreciation reserve values it seems proper to suggest that the probabilities of misclassification may be calculated for a value which is a function of several previous observations.

The presentation of the results and subsequent discussion would be remiss if suggestions to make the present simulator an even more useful tool where not included. The first suggestion would be to make the account growth factor a stochastic value. This would be more representative of the actual accounts observed than a deterministic growth factor which is presently incorporated. Secondly, there should be more than one choice of depreciation methods and procedures available. It also seems appropriate to suggest that the property account be placed on disk. Then separate routines to calculate the depreciation charges and observe the depreciation reserve behavior as well as test the sensitivity of life analysis techniques could be made.

Thirdly, calculations of the elements of the P matrix should be made more flexible. It must be remembered that the P matrix is a retirement model and that there are many positions of the matrix which have not been utilized. Fourthly, observing that there was considerable amount of calculation required to test and compute the values of the few concepts illustrated it would be a sound idea to build these routines into the program.

These aforementioned suggestions are not major programming tasks. Due to the method of construction utilized, they would be relatively easy to incorporate. These suggestions are to be considered the frosting on the cake. They were left off only because there was a degree of uncertainty as to whether or not there would even be a cake to frost. The only other programming suggestion which could not be listed under additional features is that there should be more efficient routines and computer languages which could be utilized.

CONCLUSIONS

Though the trial runs were not in depth studies, there were two significant concepts which resulted from the limited information gathered. It was the first time that the probabilities of misclassifying the mortality characteristics had been computed for a single observation of depreciation reserve. It was observed that a substantial portion of the life cycle was required before the probabilities of misclassifying became reasonably low.

The second concept was that the expected value of the depreciation reserve distribution was significantly greater than the pure value of the reserve. Previous discussion suggested that the estimated value of the depreciation reserve must be corrected upwards because of random fluctuations in the property flowthrcugh to retirement. It was also conjectured in those previous comments that the shape of the mortality distribution would be a contributing factor to the correction.

The major thrust of the research was first, finding a new technique about which a better property account generator could be built, and second, building and testing such a device. Accordingly, a retirement experience simulator was constructed whose major feature is the use of a stochastic matrix to control the property flowthrough to retirement. After several preliminary trials, a set of five specific trials was conducted. It was concluded that the property account generator worked satisfactorily. It was judged that the device has the capability to simulate property accounts not unlike those actually experienced by corporations.

This judgement was based on the capabilities built into the retirement simulator. The features are the ability to change the depreciation rate, the account growth rate, the trend percentage, the major property placements, the average property service life, and the mortality distribution individually or in groups at any desired time period. The capability to create minor fluctuations in property flowthrough is another important ability of the property account generator. Utilizing the stochastic matrix as the basis of the retirement experience simulator provided the means to distribute the effects of the changes in mortality characteristics to previously placed property.

The utilization of a stochastic matrix to control property flowthrough may be considered a third generation method to simulate property retirement experience. The flexibility of the technique places it one step beyond what has been referred to as the deterministic generator and the nondeterministic generator whose major feature was the previously describe Monte Carlo technique. The appropriate control of the elements of the stochastic matrix is what provides this flexibility.

It is expected that the techniques of controlling property flowthrough to retirement, about which the presently constructed retirement experience generator was built, will provide the basis to examine the questions concerning the comparisons, accuracy, and sensitivity of life analysis techniques and the behavior and importance of the depreciation reserve. Utilizing these techniques, it is also possible to place a property account generator as an integral and contributing subsection of a larger corporate economic model.

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APPENDIX I:

DOCUMENTATION OF THE COMPUTER PROGRAM

The appendix is divided into four major sections; program listing, flowcharts of the main program and subroutines, description of the main program and subroutines, and a list and brief description of major variables and arrays.

Section A: Program Listing

Section B: Flowcharts

Section C: Main Program and Subroutines

Section D: Major Arrays and Variables

The computer program consists of one main interlocking program and twenty-one subroutines which perform a well defined set of operations. The main program calls each subroutine in desired order. The subroutine performs the desired calculations and returns the information to the main program.

The documentation of the main program will contain a description of the major objective of each subroutine. The discussion of each subroutine will illustrate important operational details.

Section A: Program Listing

```
 B441HH JOB  'U4105,TIME=9,SIZE=192K',HOOVER
 STEP1  EXEC  WATFIV,REGION.GO=(192K,18K),TIME.GO=8
 GO.SYSIN  DD  *
JOB           U4105HOOVR,TIME=480,PAGES=30


      COMMON A(51,7), U(51,7), Y(51,3), D(51,31), WB(3,4),
     CSMRES(10), DELRES(10), SMREN(10), DELREN(10),
     C RESH(11,10), RR(31), ITP(10), DRES(50), RENTOT(51),
     CRM(20,2), SC(400), V(31), DD(31),  REN(100,10),
     C DCHRD(50), AD(50),PA(51,51),  RENH(11,10),PLANT(51),
     C RES(100,10)
      COMMON KKKK(80)
      COMMON N,FY,LY,IPA,NTR,IOPA,IOH,NPTS,E
      INTEGER RA,CA
      INTEGER RD,R,C,E,FY,LY,K1,R1,R2,  TAPE
      READ (5,55) (KKKK( M),M=1,80)
 55   FORMAT(80I1)
      READ(5,5000) NR
5000  FORMAT(I2)
      TAPE = 10
      REWIND TAPE
      DO 600 I=1,NR
      CALL ZERO
      CALL INPUT(SEED)
      CALL BUG(6)
      DO 686 IROW=1,NTR
      CALL SET1 (FLAG,RD,DIAG1)
      DO 601 II=1,E
      CALL SET2 (RD,ASL,SUCU)
      CALL DMATRX (SUCU,ASL,D,RD,U,SUCUX,ASLX,FLAG,V,TAPE,
     C RR,PA)
      CALL RANDOM(SEED,RAN)
      CALL RANPER(RM,RAN,RPER)
      DO 602 R=1,RD
      CALL SET3 (OASL,RRPER,ASL,DIAG1,RPER,RD,R)
      IF(R-1)500,501,500
 500  CONTINUE
      IF(DIAG1-DIAG2)501,502,501
 501  CONTINUE
      CALL PMATRX(WB,DIAG1)
 502  CONTINUE
      DIAG2=DIAG1
      CALL DMULTP(R ,D,DD,WB)
 602  CONTINUE
      CALL RENWL(D,RD,U,PA,V,A)
```

```
 601 CONTINUE
      CALL DEPRES(E)
      IF(IOPA) 99,99,98
98     CONTINUE
      CALL PAGE1 (I,N,FY,LY,E,IPA)
      CALL PAGE2 (E)
99     CONTINUE
      IF(IOH) 10,10,11
11     CONTINUE
      CALL HISTO1 (NPTS,IROW)
      CALL HISTO2 (NPTS,IROW)
10     CONTINUE
686    CONTINUE
      IF(IOH) 89,89,88
88     CONTINUE
      CALL HISAGN (NPTS,NTR,RES,SMRES,DELRES,RESH)
      CALL HISAGN (NPTS,NTR,REN,SMREN,DELREN,RENH)
      CALL PAGE1 (I,N,FY,LY,E,IPA)
      CALL PAGE3
89     CONTINUE
 600 CONTINUE
      STOP
      END


      SUBROUTINE MUSIG
      COMMON A(51,7), U(51,7), Y(51,3), D(51,31), WB(3,4),
    CSMRES(10), DELRES(10), SMREN(10), DELREN(10),
    C RESH(11,10), RR(31), ITP(10), DRES(50), RENTOT(51),
    CRM(20,2), SC(400), V(31), DD(31),  REN(100,10),
    C DCHRD(50), AD(50),PA(51,51),  RENH(11,10),PLANT(51),
    C RES(100,10)
      COMMON KKKK(80)
      COMMON N,FY,LY,IPA,NTR,IOPA,IOH,NPTS,E
      INTEGER RA,CA
     INTEGER RD,R,C,E,FY,LY,K1,R1,R2,  TAPE
      DO 1 K=1,NPTS
      SUM=0
      DO 2 J=1,NTR
      SUM=SUM+RES(J,K)
  2   CONTINUE
      ZMEAN=SUM/NTR
      RES(NTR+1,K)=ZMEAN
      SUM=0
      DO 3 J=1,NTR
      SUM=SUM +(RES(J,K)-ZMEAN)*(RES(J,K)-ZMEAN)
  3   CONTINUE
      ZVAR=SUM/(NTR-1)
      RES(NTR+2,K) = ZVAR
```

```
      ZNTR=NTR
      RES(NTR+3,K) = SQRT(ZVAR)   / SQRT(ZNTR)
 1    CONTINUE
      RETURN
      END


     SUBROUTINE INPUT(SEED)
      COMMON A(51,7), U(51,7), Y(51,3), D(51,31), WB(3,4),
    CSMRES(10), DELRES(10), SMREN(10), DELREN(10),
    C RESH(11,10), RR(31), ITP(10), DRES(50), RENTOT(51),
    CRM(20,2), SC(400), V(31), DD(31),  REN(100,10),
    C DCHRD(50), AD(50),PA(51,51),  RENH(11,10),PLANT(51),
    C RES(100,10)
      COMMON KKKK(80)
      COMMON N,FY,LY,IPA,NTR,IOPA,IOH,NPTS,E
      INTEGER RD,R,C,E,FY,LY,K1,R1,R2,TAPE,ONE,RA,CA,RRM,
    C CV,CD,RDN,FCA,
    CRY,CY
C****READ FIRST CARD OF SET-ACCOUNT RANGE
      READ(5,5001) FY,LY,IPA,NTR,IOPA,IOH,NPTS,(ITP(I),I=1
    C ,NPTS)
      CALL BUG(1)
      SEED =IPA
     E=LY-FY+1
     YY=FY
     DO 9 R=1,E
     A(R,1)=YY
     U(R,1)=YY
     YY=YY+1
   9 CONTINUE
C****READ SECOND CARD OF SET-PLACEMENTS
     READ(5,5002) N,((Y(R,C),C=1,2),R=1,N)
      CALL BUG(2)
     CALL LOAD(Y,A,FY,E,U)
C****READ THIRD CARD OF SET-SURVIVOR CURVES
     READ(5,5003) N,((Y(R,C),C=1,3),R=1,N)
      CALL BUG(3)
     CALL CHANGE(2,Y,A,FY,3,E)
C****READ FOURTH CARD OF SET-GROWTH PATTERNS
     READ(5,5004) N,((Y(R,C),C=1,2),R=1,N)
      CALL BUG(4)
     CALL CHANGE(1,Y,A,FY,5,E)
C****READ FIFTH CARD OF SET-TREND PATTERNS
     READ(5,5004) N,((Y(R,C),C=1,2),R=1,N)
      CALL BUG(5)
     CALL CHANGE(1,Y,A,FY,6,E)
C****READ 5.5 CARD OF SET-DEPRECIATION RATE
     READ(5,5004) N,((Y(R,C),C=1,2),R=1,N)
```

```
      CALL CHANGE(1,Y,A,FY,7,E)
C****READ SIXTH CARD OF SET-P(R,C) RANDOMIZATION
      READ(5,5004) N,((RM(R,C),C=1,2),R=1,N)
       CALL BUG(6)
5001    FORMAT(  3(I4,1X),I2,2(1X,I1),1X,11(I2,1X))
5002 FORMAT(I2,1X,100(F5.0,F7.0,2X))
5003 FORMAT(I2,1X,100(F5.0,F3.0,F4.1,2X))
5004 FORMAT(I2,1X,100(F5.0,F5.1))
       RETURN
       END


       SUBROUTINE PAGE1  (I,N,FY,LY,E,IPA)
       COMMON A(51,7), U(51,7), Y(51,3), D(51,31), WB(3,4),
     CSMRES(10), DELRES(10), SMREN(10), DELREN(10),
     C RESH(11,10), RR(31), ITP(10), DRES(50), RENTOT(51),
     CRM(20,2), SC(400), V(31), DD(31),  REN(100,10),
     C DCHRD(50), AD(50),PA(51,51),  RENH(11,10),PLANT(51),
     C RES(100,10)
       INTEGER RD,R,C,E,FY,LY,K1,R1,R2,TAPE,ONE,RA,CA,RRM,
     C CV,CD,RDN,FCA,
     CRY,CY
      WRITE(6,6000)IPA,FY,LY
      IA3=A(1,3)
      IA1=A(1,1)
      WRITE(6,6001)IA1,IA3,(A(1,J),J=4,7)
      DO 803 L=2,E
       IF(A(L-1,7) - A(L,7))804,799,804
 799 IF(A(L-1,3) -A(L,3))804,800,804
 800 IF(A(L-1,4) -A(L,4))804,801,804
 801 IF(A(L-1,5) -A(L,5))804,802,804
 802 IF(A(L-1,6) -A(L,6))804,803,804
 804 CONTINUE
      IA1=A(L,1)
      IA3=A(L,3)
      WRITE(6,6002)IA1,IA3,(A(L,J),J=4,7)
 803 CONTINUE
       WRITE(6,6009)   (RM(R,2), R=1,N)
       WRITE(6,6010)   (RM(R,1),R=1,N)
       RETURN
6000 FORMAT('1ACCOUNT ',I4,2X,'FROM',I5,' -',I5,//,8X,
     C 'YEAR',5X,'DISTRIBUTION',5X,'GROWTH',6X,'TREND'
     C ,'DEPRECIATION RATE')
6001 FORMAT('0',7X,I4,7X,I2,2X,F4.1,7X,F4.1,'%',6X,F5.1,'%'
     C ,10X,F9.1,'%')
6002 FORMAT(8X,I4,7X,I2,2X,F4.1,7X,F4.1,'%',6X,F5.1,'%',1
     C0X,F9.1,'%')
6009   FORMAT (' ',///,' ',10X,'RANDOM DISTRIBUTION',/,' ',
     C 12X,'VARIATION',5X,20F6.1)
```

```
6010    FORMAT      (' ',12X,'P(VARIATION)',2X,20F6.1)
      END


      SUBROUTINE DEPRES(E)
      INTEGER E,ONE
      ONE=1
      PLANT(ONE)= U(ONE,2)
      RENTOT(ONE) = U(ONE,7)
      DCHRD(ONE) = (U(ONE,2))*( 2*A(ONE,7))/400
      AD(ONE) = DCHRD(ONE)
      DRES(ONE) = AD(ONE) - RENTOT(ONE)
      DO 3 K=2  ,E
      PLANT(K) = PLANT(K-ONE) + U(K,2) + U(K,4)
      DCHRD(K) = (PLANT (K) + PLANT(K-ONE))* (2*A(K,7))/400
      RENTOT(K) = RENTOT(K-ONE) + U(K,7)
      AD(K) = AD(K-ONE) + DCHRD(K)
      DRES(K) = AD(K) - RENTOT(K)
3       CONTINUE
      CALL BUG(7)
      RETURN
      END


      SUBROUTINE LOAD(Y,A,FY,E,U)
      INTEGER R,E,RY,       C,FY
      DIMENSION Y(51,3),A(51,7),  U(51,7)
      CKY=FY
      DO 20 R=1,E
      A(R,1)=CKY
      U(R,1)=CKY
      CKY=CKY+1
   20 CONTINUE
      RY=1
      CKY=Y(RY,1)
      DO 21 R=1,E
      IF(CKY-A(R,1))23,22,23
   22 CONTINUE
      A(R,2)=Y(RY,2)
      U(R,2)=Y(RY,2)
      RY=RY+1
      CKY=Y(RY,1)
      GO TO 21
   23 CONTINUE
      A(R,2)=0
      U(R,2)=0
   21 CONTINUE
      DO 18 R=1,51
      DO 18 C=1,3
```

```
18 Y(R,C)=0
   RETURN
   END


   SUBROUTINE CHANGE(N,Y,A,FY,FCA,E)
   INTEGER FCA,E,RY,CA,CY,R,C,RA,FY
   DIMENSION A(51,7),Y(51,3)
   CKY=FY
   RY=1
   CA=FCA-1
   CY=1
   DO 12 I=1,N
   CA=CA+1
   CY=CY+1
   A(1,CA)=Y(RY,CY)
12 CONTINUE
   DO 13 RA =2,E
   CA=FCA-1
   CKY=CKY+1
   CY=1
   IF(Y(RY+1,1)-CKY) 14,15,14
15 CONTINUE
   RY=RY+1
   DO 16 I=1,N
   CA=CA+1
   CY=CY+1
   A(RA,CA)=Y(RY,CY)
16 CONTINUE
   GO TO 13
14 DO 17 I=1,N
   CA=CA+1
   CY=CY+1
   A(RA,CA)=Y(RY,CY)
17 CONTINUE
13 CONTINUE
   DO 18 R=1,51
   DO 18 C=1,3
18 Y(R,C)=0
   RETURN
   END


    SUBROUTINE ZERO
    COMMON A(51,7), U(51,7), Y(51,3), D(51,31), WB(3,4),
  CSMRES(10), DELRES(10), SMREN(10), DELREN(10),
  C RESH(11,10), RR(31), ITP(10), DRES(50), RENTOT(51),
  CRM(20,2), SC(400), V(31), DD(31),  REN(100,10),
  C DCHRD(50), AD(50),PA(51,51),  RENH(11,10),PLANT(51),
```

```
C RES(100,10)
   INTEGER RD,R,C,E,FY,LY,K1,R1,R2,TAPE,ONE,RA,CA,RRM,
C CV,CD,RDN,FCA,
CRY,CY
  DO 1 R=1,51
  DO 2 C=1,7
2 A(R,C)=0
  DO 3 C=1,7
3 U(R,C)=0
  DO 4 C=1,3
4 Y(R,C)=0
  DO 5 C=1,31
   RR(C)=0
  V(C)=0
  DD(C)=0
5 D(R,C)=0
  DO 6 C=1,51
6 PA(R,C)=0
1 CONTINUE
  DO 7 R=1,20
  DO 7 C=1,2
7 RM(R,C)=0
      DO 334 R=1,3
      DO 334 C=1,4
334    WB(R,C)=0
      DO 331 C=1,10
      ITP(C)     =0
      SMRES(C)       =0
      DELRES(C)     =0
      SMREN(C)       =0
      DELREN(C)      =0
      DO 332 R=1,11
      RENH(R,C)     =0
      RESH(R,C)      =0
332    CONTINUE
      DO 333 J=1,100
      REN(J,C)              =0
      RES(J,C)           =0
333    CONTINUE
331    CONTINUE
      DO 335 R=1,50
      DRES(R)  =0
      RENTOT(R)=0
      DCHRD(R)=0
      AD(R)=0
335    CONTINUE
      RETURN
      END
```

```
      SUBROUTINE SET1 (FLAG,RD,DIAG1)
      COMMON A(51,7), U(51,7), Y(51,3), D(51,31), WB(3,4),
    CSMRES(10), DELRES(10), SMREN(10), DELREN(10),
    C RESH(11,10), RR(31), ITP(10), DRES(50), RENTOT(51),
    CRM(20,2), SC(400), V(31), DD(31),  REN(100,10),
    C DCHRD(50), AD(50),PA(51,51),  RENH(11,10),PLANT(51),
    C RES(100,10)
      INTEGER RD
     RD=0
     U(1,5)=U(1,2)
     PA(1,1)=U(1,5)
     FLAG=0
     DIAG1=1000
      RETURN
      END


      SUBROUTINE DMATRX(SUCU,ASL,D,RD,U,SUCUX,ASLX,FLAG,
    CV,TAPE,RR,PA)
        DIMENSION RR(31),U(51,7),V(21),D(51,31),PA(51,51),
    CSC(400)
     IF(FLAG)31,30,31
  31 CONTINUE
     IF(ASL-ASLX)30,32,30
  32 CONTINUE
     IF(SUCU-SUCUX)30,33,30
  30 CONTINUE
     FLAG=1
     IF(SUCU-1)34,35,34
  34 K=SUCU-1
     DO 36 I=1,K
C****READ DUMMY FOR POSITIONING
     READ(TAPE)(DUMMY,J=1,400)
 35    CONTINUE
 36   CONTINUE
C****READ IOWA SURVIVOR CURVE
     READ(TAPE)(SC(IC),IC=1,400)
     REWIND TAPE
      DO 157 I=ONE,31
      V(I)=0
      RR(I)=0
157    CONTINUE
     Q=0.5
     C=1
     Q1=SC(C)
     OAGE=0
     CV=0
     P=0.01
```

```
   38 CONTINUE
      QD=0
   39 CONTINUE
      AGE=P*ASL
      IF(AGE-Q) 40,41,41
   40 CONTINUE
      C=C+1
      Q2=SC(C)
      QD=QD+Q1-Q2
      P=P+0.01
      Q1=Q2
      OAGE=AGE
      GO TO 39
   41 CONTINUE
      PC=(Q-OAGE)/(AGE-OAGE)
      Q2=SC(C+1)
      QP=(Q1-Q2)*PC
      CV=CV+1
      V(CV)=QD+QP
      Q1=Q1-QP
      Q=Q+1
      IF(Q1)43,43,38
   43 CONTINUE
      SUCUX=SUCU
      ASLX=ASL
       Z=ONE-V(ONE)
       DO 156 I=ONE,30
       RR(I)=V(I+ONE)/Z
156     CONTINUE
   33 CONTINUE
       IF(RD-ONE)  158,158,159
158     CONTINUE
       U(ONE,5)=U(ONE,2)
       U(ONE,3)=0
       U(ONE,4)=0
       U(ONE,6)=U(ONE,5)*V(ONE)
       U(ONE,7)=U(ONE,6)
       PA(ONE,ONE)=U(ONE,5)
159     CONTINUE
       DO 160 I=ONE,30
       D(RD,I+ONE)=RR(I)*U(RD,5)
160     CONTINUE
      RETURN
      END


      SUBROUTINE RANDOM(SEED,RAN)
      X=SQRT(SEED)
      XX=10*X
```

```
      IX=XX
      DX=XX-IX
      SEED=10000*DX
      IF(SEED-1)41,41,42
   41 CONTINUE
      SEED=SEED+3.356
   42 CONTINUE
      XX=1000*X
      NX=XX
      RAN=XX-NX
      RETURN
      END


      SUBROUTINE RANPER(RM,RAN,RPER)
      DIMENSION RM(20,2)
      INTEGER RRM
      RRM=0
      PS=0
   60 CONTINUE
      RRM=RRM+1
      CALL BUG(6)
      PS=PS+RM(RRM,1)/1000
      IF(PS-RAN)60,61,61
   61 CONTINUE
      RPER=RM(RRM,2)/100
      RETURN
      END


       SUBROUTINE  SET2(RD,ASL,SUCU)
       COMMON A(51,7), U(51,7), Y(51,3), D(51,31), WB(3,4),
     CSMRES(10), DELRES(10), SMREN(10), DELREN(10),
     C RESH(11,10), RR(31), ITP(10), DRES(50), RENTOT(51),
     CRM(20,2), SC(400), V(31), DD(31),  REN(100,10),
     C DCHRD(50), AD(50),PA(51,51),  RENH(11,10),PLANT(51),
     C RES(100,10)
       INTEGER RD
      RD=RD+1
      ASL=A(RD,4)
      SUCU=A(RD,3)
       RETURN
       END


       SUBROUTINE SET3 (OASL,RRPER,ASL,DIAG1,RPER,RD,R)
       COMMON A(51,7), U(51,7), Y(51,3), D(51,31), WB(3,4),
     CSMRES(10), DELRES(10), SMREN(10), DELREN(10),
     C RESH(11,10), RR(31), ITP(10), DRES(50), RENTOT(51),
```

```
     CRM(20,2), SC(400), V(31), DD(31),  REN(100,10),
     C DCHRD(50), AD(50),PA(51,51),  RENH(11,10),PLANT(51),
     C RES(100,10)
       INTEGER RD,R
      OASL=A(R,4)
      RRPER=(OASL-ASL)/ASL
      DIAG1=RPER+RRPER+A(RD,6)/100
       RETURN
       END


       SUBROUTINE PMATRX(WB,DIAG1)
       DIMENSION WB(3,4)
       IF(DIAG1) 71,73,74
71      CONTINUE
       A=-DIAG1
       B=1-A
       DO 72 I=1,3
       WB(I,I)=B
       WB(I,I+1)=A
72      CONTINUE
       WB(2,1)=0
       WB(3,2)=0
       GO TO 78
74      CONTINUE
       A=DIAG1
       B=1-DIAG1
       DO 75 I=1,3
       WB(I,I+1)=0
75      CONTINUE
       WB(1,1)=1
       DO 76 I=1,2
       WB(I+1,I)=A
       WB(I+1,I+1)=B
76      CONTINUE
       GO TO 78
73       CONTINUE
       DO 77 I=1,3
       WB(I,I)=1
       WB(I,I+1)=0
77      CONTINUE
       WB(2,1)=0
       WB(3,2)=0
78      CONTINUE
       RETURN
       END


       SUBROUTINE DMULTP(RD,D,DD,WB)
```

```
      DIMENSION D(51,31),DD(31), WB(3,4)
      INTEGER RD
      DD(1)=WB(1,1)*D(RD,2)+WB(2,1)*D(RD,3)
      DO 50 I=2,29
      DD(I)= WB(1,2)*D(RD,I) + WB(2,2)*D(RD,I+1) +
     C WB(3,2)*D(RD,I+2)
50     CONTINUE
      DD(30)=WB(2,3)*D(RD,30)+WB(3,3)*D(RD,31)
      DD(31)=WB(3,4)*D(RD,31)
      DO 51 I=1,31
      D(RD,I)=DD(I)
      DD(I)=0
51    CONTINUE
      RETURN
      END


      SUBROUTINE RENWL(D,RD,U,PA,V,A)
      DIMENSION D(51,31),U(51,7),PA(51,51),V(31),A(51,5)
      INTEGER RD, RDN, ONE
      ONE=1
      RDN=ONE+RD
      SM=0
      SMM=0
      DO 2 I=ONE, RD
      SM=D(I,ONE)+SM
      SMM = U(I,2) + SMM + U(I,4)
2      CONTINUE
      U(RDN,3)=SM
      U(RDN,4)=SMM*A(RD,5)/100
      U(RDN,5)= U(RDN,2)+U(RDN,3)+U(RDN,4)
      U(RDN,6)= V(ONE)*U(RDN,5)
      U(RDN,7)=U(RDN,3)+U(RDN,6)
      PA(RDN,RDN)=U(RDN,5)
      DO 3 I=ONE,RD
      PA(I,RDN)=PA(I,RD)-D(I,1)
3      CONTINUE
      RETURN
      END


      SUBROUTINE PAGE2 (E)
      COMMON A(51,7), U(51,7), Y(51,3), D(51,31), WB(3,4),
     CSMRES(10), DELRES(10), SMREN(10), DELREN(10),
     C RESH(11,10), RR(31), ITP(10), DRES(50), RENTOT(51),
     CRM(20,2), SC(400), V(31), DD(31), REN(100,10),
     C DCHRD(50), AD(50),PA(51,51), RENH(11,10),PLANT(51),
     C RES(100,10)
      INTEGER RD,R,C,E,FY,LY,K1,R1,R2,TAPE,ONE,RA,CA,RRM,
```

```
     C CV,CD,RDN,FCA,
     CRY,CY
       K=1
812     CONTINUE
       IZP=17*K
       R1=IZP-16
       IF(E-IZP)810,810,811
810     CONTINUE
       R2=E-(K-1)*17  +R1-1
       CALL PRNNT(R1,R2,E)
       RETURN
811     CONTINUE
       R2=IZP
       CALL PRNNT(R1,R2,E)
       K=K+1
       GO TO 812
       END


       SUBROUTINE PRNNT(R1,R2,E)
       COMMON A(51,7), U(51,7), Y(51,3), D(51,31), WB(3,4),
     CSMRES(10), DELRES(10), SMREN(10), DELREN(10),
     C RESH(11,10), RR(31), ITP(10), DRES(50), RENTOT(51),
     CRM(20,2), SC(400), V(31), DD(31),  REN(100,10),
     C DCHRD(50), AD(50),PA(51,51),  RENH(11,10),PLANT(51),
     C RES(100,10)
       INTEGER R1,R2,E,R,C
      WRITE(6,6003) (U(K1,1),K1=R1,R2)
      WRITE(6,6004) (U(K1,2),K1=R1,R2)
      WRITE(6,6005) (U(K1,3),K1=R1,R2)
      WRITE(6,6006) (U(K1,4),K1=R1,R2)
      WRITE(6,6007) (U(K1,5),K1=R1,R2)
       WRITE(6,6009)   (U(K1,6),K1=R1,R2)
       WRITE(6,6010)   (U(K1,7),K1=R1,R2)
       WRITE(6,6011)  ( PLANT(K1), K1=R1,R2)
       WRITE(6,6012)  ( DCHRD(K1)  ,K1 = R1,R2)
       WRITE(6,6013)  ( AD(K1), K1 = R1,R2)
       WRITE(6,6014)  ( RENTOT(K1), K1 = R1,R2)
       WRITE(6,6015)  (DRES(K1),K1 = R1, R2)
      DO 807 C=1,E
       IK=U(C,1)
      WRITE(6,6008)IK     ,(PA(R,C),R=R1,R2)
 807  CONTINUE
6003  FORMAT('1YEAR',5X,17F7.0,/,' PLACEMENT')
6004  FORMAT('  INITIAL',1X,17(F7.0 ))
6005  FORMAT('  RENEWAL',1X,17(F7.0 ))
6006  FORMAT('  GROWTH',2X,17(F7.0 ))
6007   FORMAT('  INPUT',3X,17(F7.0))
6008  FORMAT(I5,5X,17(F7.0 ))
```

```
6009    FORMAT('  ADJUST',2X,17(F7.0))
6010    FORMAT('  RETIRE',2X,17(F7.0),/,' YEAR END')
6011     FORMAT('  PLANT   ',17(F7.0))
6012     FORMAT('  DCHRD   ',17(F7.0))
6013     FORMAT('  AD      ',17(F7.0))
6014     FORMAT('  RENTOT  ',17(F7.0))
6015     FORMAT('  DRES    ',17(F7.0))
      RETURN
      END


      SUBROUTINE HISTO1 (NPTS,IROW)
      COMMON A(51,7), U(51,7), Y(51,3), D(51,31), WB(3,4),
     CSMRES(10), DELRES(10), SMREN(10), DELREN(10),
     C RESH(11,10), RR(31), ITP(10), DRES(50), RENTOT(51),
     CRM(20,2), SC(400), V(31), DD(31),  REN(100,10),
     C DCHRD(50), AD(50),PA(51,51),  RENH(11,10),PLANT(51),
     C RES(100,10)
      DO 3 K= 1,NPTS
      IC= ITP(K)
      REN(IROW,K)=U(IC,7)
  3   CONTINUE
      RETURN
      END


      SUBROUTINE HISTO2 (NPTS,IROW)
      COMMON A(51,7), U(51,7), Y(51,3), D(51,31), WB(3,4),
     CSMRES(10), DELRES(10), SMREN(10), DELREN(10),
     C RESH(11,10), RR(31), ITP(10), DRES(50), RENTOT(51),
     CRM(20,2), SC(400), V(31), DD(31),  REN(100,10),
     C DCHRD(50), AD(50),PA(51,51),  RENH(11,10),PLANT(51),
     C RES(100,10)
      DO 3 K= 1,NPTS
      IC= ITP(K)
      RES(IROW,K)= DRES(IC)
  3   CONTINUE
      RETURN
      END


      SUBROUTINE HISAGN (NPTS,NTR,DI,SM,DELTA,HI)
      DIMENSION DI(100,10),SM(10), DELTA(10), HI(11,10)
      INTEGER ONE
      ONE=1
       DO 8 K=ONE,NPTS
      SM(K) = DI(ONE,K)
      BG = DI(ONE,K)
      DO 4 J = ONE,NTR
```

```
      IF ( SM(K) - DI(J,K)) 6,6,5
5      CONTINUE
      SM(K) = DI(J,K)
6      CONTINUE
      IF (BG-DI(J,K)) 7,4,4
7      CONTINUE
      BG = DI(J,K)
4      CONTINUE
      DELTA (K) = (BG -SM(K)+1)/11
      PL = SM(K) - ONE
      PH = SM(K) + DELTA(K)
      DO 8 JJ= ONE,11
      HI(JJ,K) =0
      DO 11 J=ONE,NTR
      IF(PL - DI(J,K)) 10,11,11
10     CONTINUE
      IF (DI(J,K) - PH) 12,12,11
12     CONTINUE
      HI(JJ,K) = HI(JJ,K) + ONE
11     CONTINUE
      PL = PH
      PH = PH + DELTA(K)
8     CONTINUE
      DO 13 K=ONE,NPTS
      DO 13 JJ = ONE, 11
      HI(JJ,K) = HI (JJ,K)/ NTR
13     CONTINUE
      RETURN
      END


      SUBROUTINE BUG(K)
      COMMON A(51,7), U(51,7), Y(51,3), D(51,31), WB(3,4),
     CSMRES(10), DELRES(10), SMREN(10), DELREN(10),
     C RESH(11,10), RR(31), ITP(10), DRES(50), RENTOT(51),
     CRM(20,2), SC(400), V(31), DD(31), REN(100,10),
     C DCHRD(50), AD(50),PA(51,51), RENH(11,10),PLANT(51),
     C RES(100,10)
      COMMON KKKK(80)
      COMMON N,FY,LY,IPA,NTR,IOPA,IOH,NPTS,E
      INTEGER FY,LY,E,R,C
      IF (KKKK(K)) 50,50,51
50     RETURN
51      CONTINUE
      GO TO(1,2,3,4,5,6,7,8,9),K
1      CONTINUE
     WRITE(6,402)K,FY,LY,IPA,NTR,IOPA,IOH,NPTS,(ITP(I),I=1
     C,NPTS)
      RETURN
```

```
2       CONTINUE
      WRITE(6,81) K,N, ((Y(R,C) ,C=1,2) ,R=1,N)
       RETURN
3       CONTINUE
      WRITE(6,81) K,N, ((Y(R,C) ,C=1,3) ,R=1,N)
       RETURN
4       CONTINUE
      WRITE(6,81) K,N, ((Y(R,C) ,C=1,2) ,R=1,N)
       RETURN
5       CONTINUE
      WRITE(6,81) K,N, ((Y(R,C) ,C=1,2) ,R=1,N)
       RETURN
6       CONTINUE
       WRITE (6,81 ) K,N, ((RM(R,C) ,C=1,2) ,R=1,N)
       RETURN
7       CONTINUE
       WRITE(6,82)  (PLANT(J) ,J=1,5), (DCHRD(J) ,J=1,5) ,
     C(RENTOT(J) ,J=1
     C,5), (AD(J), J=1,5) , (DRES(J) ,J=1,5)
       RETURN
8       CONTINUE
       RETURN
9       CONTINUE
       RETURN
81      FORMAT (' CARD' ,I1,I4,10F10.1)
402    FORMAT(' CARD',I1, 2X,3I5,14I3)
82     FORMAT(' PLANT ', 5F10.1,/,' DCHRD ',5F10.1,/,'
     CRENTOT' 5F10.1,/,
     C' AD   ', 5F10.1, /,' DRES ',5F10.1 )
         END


       SUBROUTINE PAGE3
       DIMENSION RGNRES(12,10), RGNREN(12,10), YEAR(10)
       COMMON A(51,7), U(51,7), Y(51,3), D(51,31), WB(3,4),
     CSMRES(10), DELRES(10), SMREN(10), DELREN(10),
     C RESH(11,10), RR(31), ITP(10), DRES(50), RENTOT(51),
     CRM(20,2), SC(400), V(31), DD(31), REN(100,10),
     C DCHRD(50), AD(50),PA(51,51),  RENH(11,10),PLANT(51),
     C RES(100,10)
       COMMON KKKK(80)
       COMMON N,FY,LY,IPA,NTR,IOPA,IOH,NPTS,E
       INTEGER RA,CA
      INTEGER RD,R,C,E,FY,LY,K1,R1,R2,  TAPE
       INTEGER ONE
       ONE = 1
       DO 3  K = ONE,NPTS
       DO 3  J = 1,12
       RGNRES(J,K) = SMRES(K) + (J-1)*DELRES(K)
```

```
      RGNREN(J,K) = SMREN(K) + (J-1)*DELREN(K)
3      CONTINUE
      DO 4 K= ONE,NPTS
      YEAR(K) = ITP (K) - ONE + FY
4      CONTINUE
      DO 5 K = ONE,NPTS
      WRITE(6,22) YEAR(K), (RGNRES(J,K), J=1,11),
    C(RGNRES(J,K), J=2,12
    C), (RESH(J,K) ,J=1,11)
     WRITE(6,23) (RGNREN(J,K),J=1,11),(RGNREN(J,K),J=2,12
    C), (RENH(J,K) ,J=1,11)
5     CONTINUE
      WRITE(6,40)
      WRITE(6,41)
      WRITE(6,42) (ITP(K),K=1,NPTS)
      DO 44 J=1,NTR
      WRITE(6,43) (RES(J,K),K=1,NPTS)
44     CONTINUE
      CALL MUSIG
      WRITE(6,45) (RES(NTR+1,K) , K=1,NPTS)
      WRITE(6,46) (RES(NTR+2,K) , K=1,NPTS)
      WRITE(6,47) (RES(NTR+3,K) , K=1,NPTS)
 40    FORMAT('1')
 41   FORMAT(' DEPRECIATION RESERVE')
 42    FORMAT(' YEAR',20I9)
 43    FORMAT(' ',4X,20F9.2)
 47    FORMAT(' STDP',20F9.2)
 46    FORMAT(' VARS',20F9.2)
 45    FORMAT(' MEAN',20F9.2)
22    FORMAT(' ',//,' TEST YEAR ',F6.0,/,'
    C DEPRECIATION RESERVE',//,
    C' RANGE ',11F9.2,/,7X,11F9.2,/,'   %    ',11F9.2)
    C/,'   %    ',11F9
23    FORMAT('ORENEWALS',/,' RANGE ',11F9.2,/,7X,11F9.2,
    C.2)
      RETURN
      END
```

```
ENTRY
2
1961 1980 5573 30 0 1 10-11-12-13-14-15-16-17-18-19-20
01 1961. 10000.
01 1961. 8. 5.
01 1961.  0.
02 1961.  0. 1970.  5.
01 1961. 20.
5  50. -5.  200. -3.  500.  0.  200.  3.  50.  5.
1961 1980 7573 30 0 1 10-11-12-13-14-15-16-17-18-19-20
```

```
01 1961. 10000.
01 1961. 8. 5.
01 1961.  0.
02 1961.  0. 1970.  7.
01 1961. 20.
5   50. -5.  200. -3.  500.  0.  200.  3.   50.  5.
STOP  GO.FT10F001  DD
UNIT=TAPE,DISP=(OLD,KEEP),LABEL=(,NL,,IN),
              DCB=(TRTCH=C,DEN=2),VOLUME=SER=TP0541
```

Section B: Flowcharts

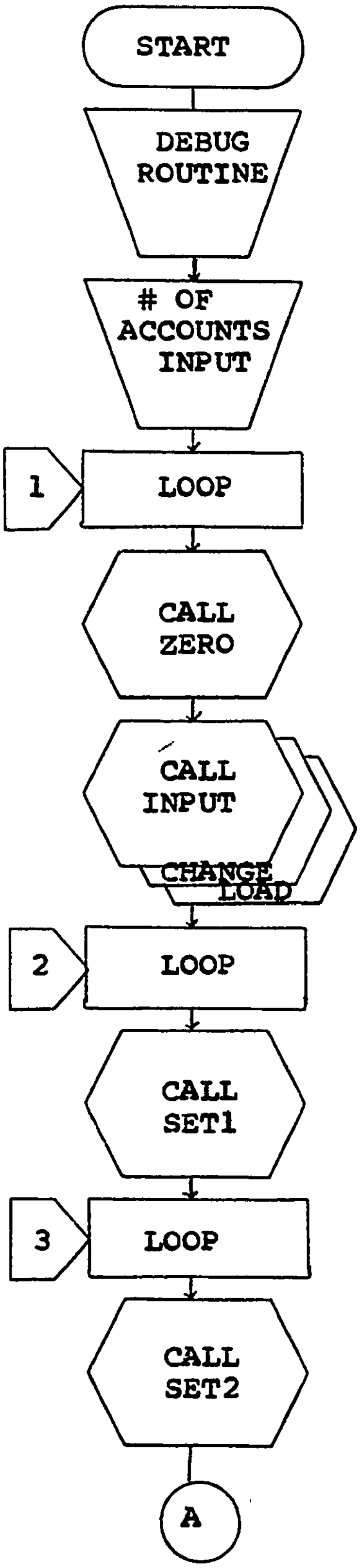


Figure 11. Main Program

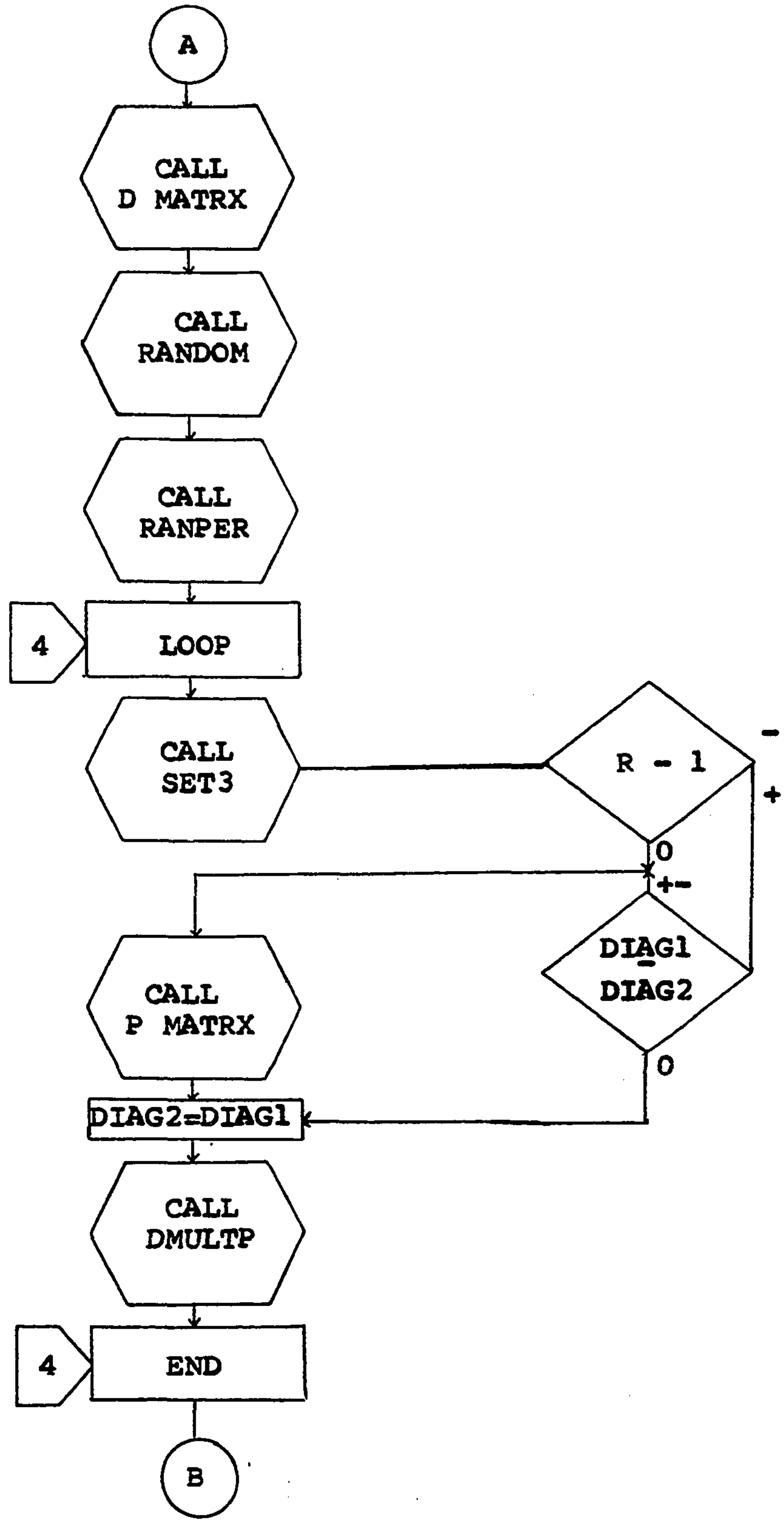


Figure 11. (continued)

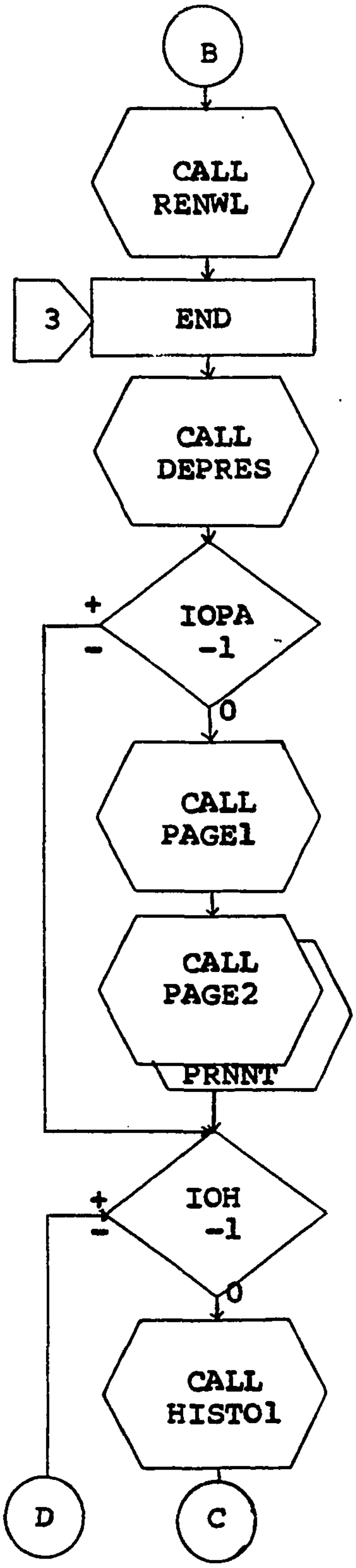


Figure 11. (continued)

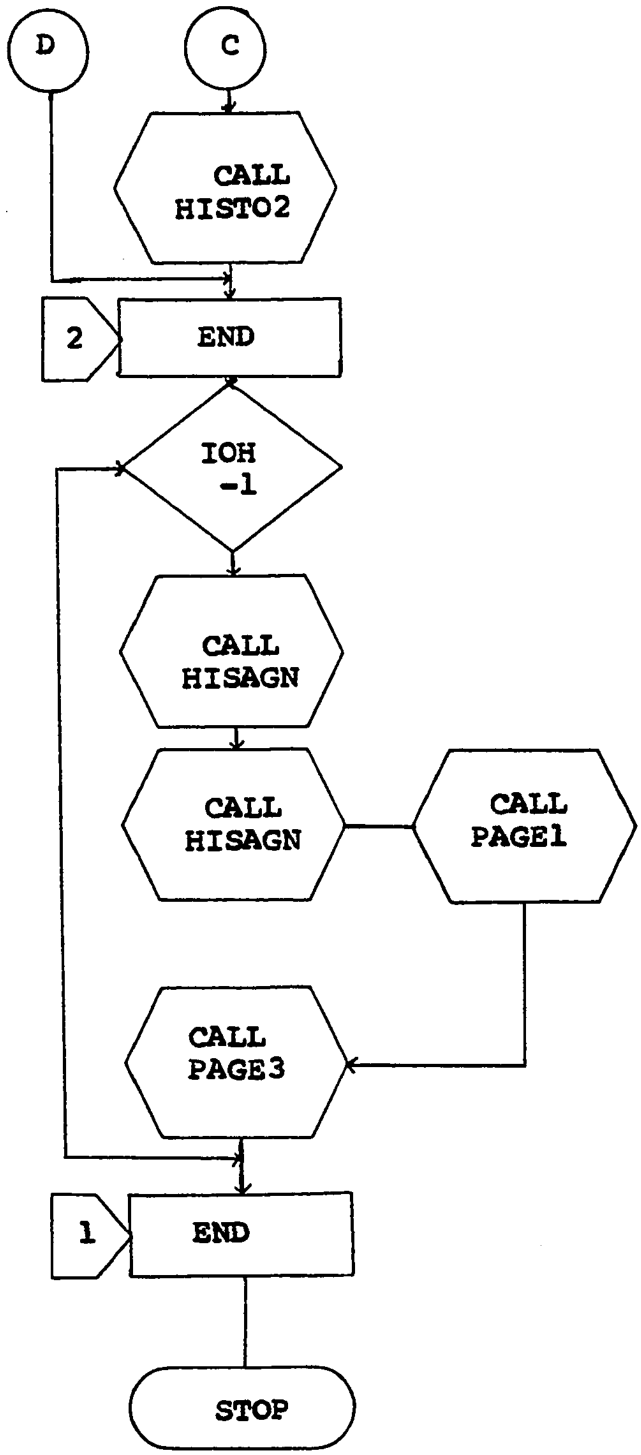


Figure 11. (continued)

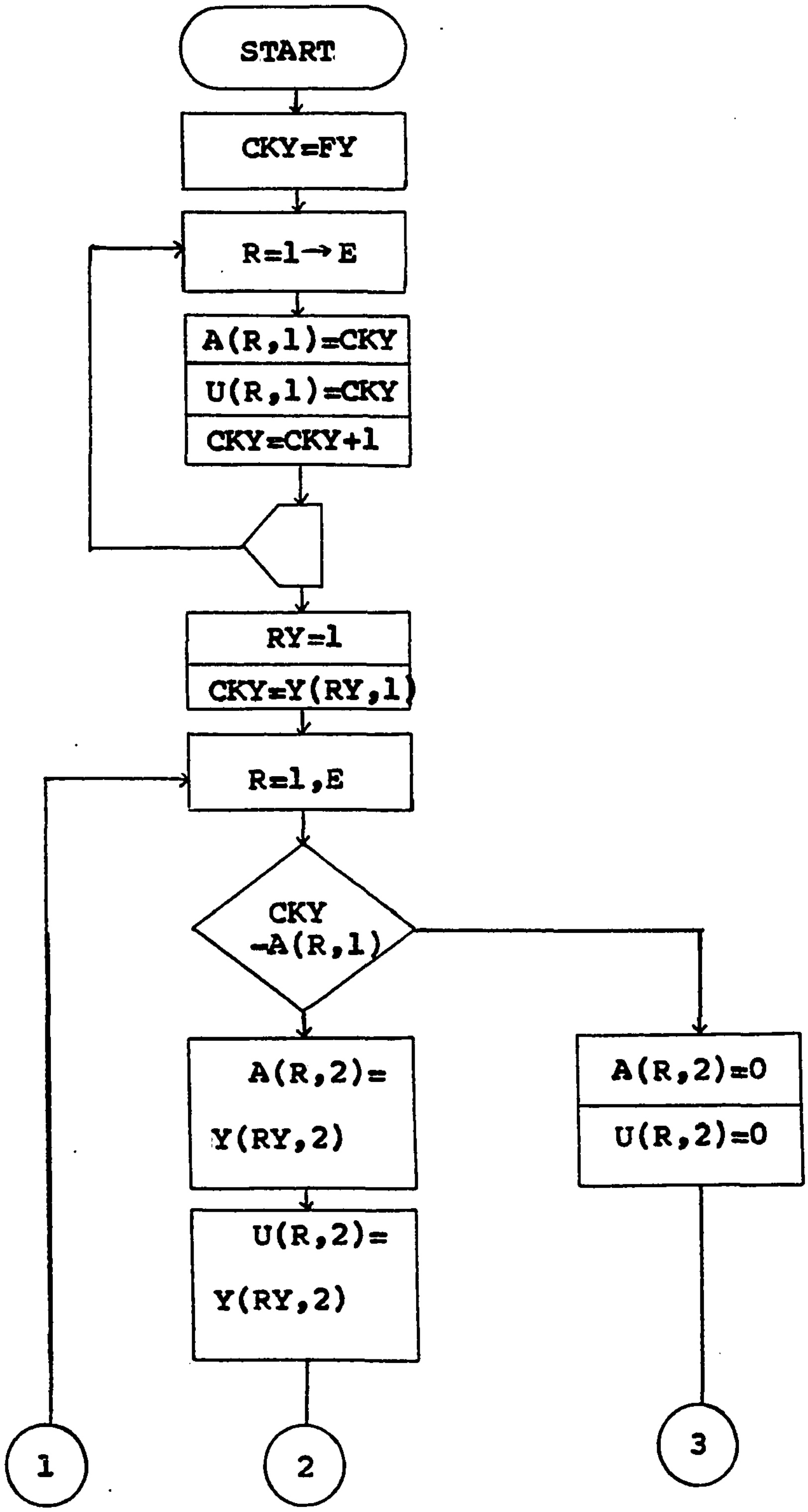


Figure 12. LOAD Subroutine

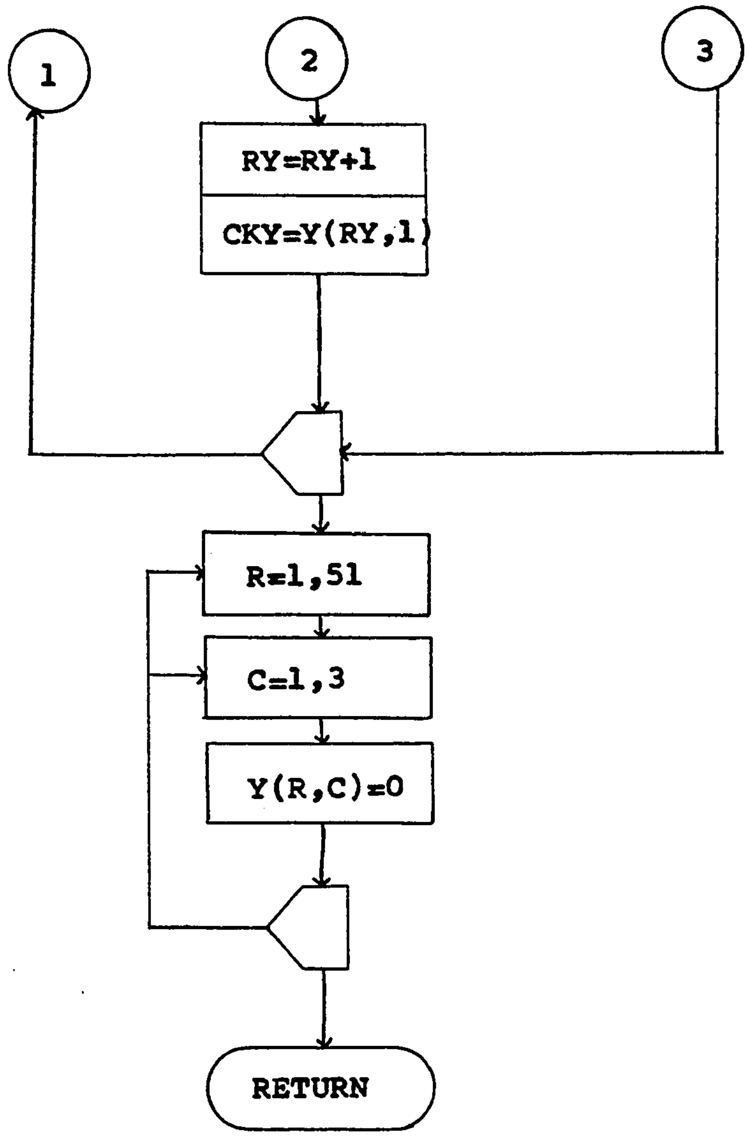


Figure 12. (continued)

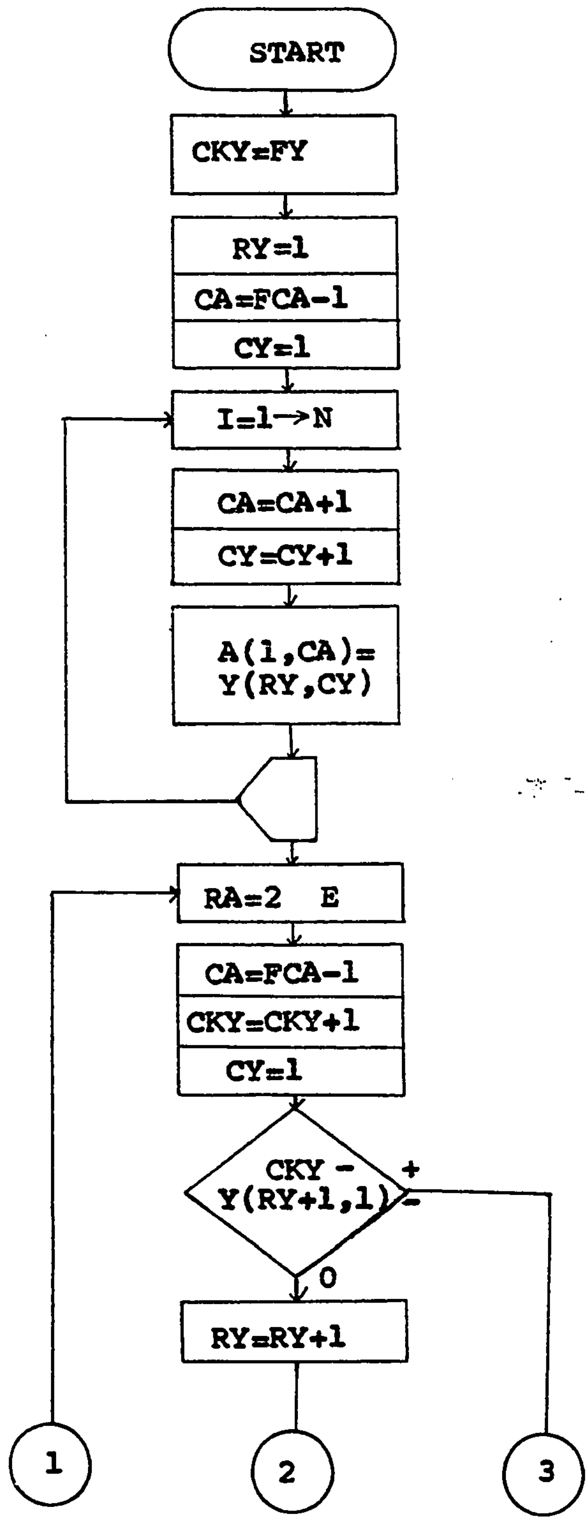


Figure 13. CHANGE Subroutine

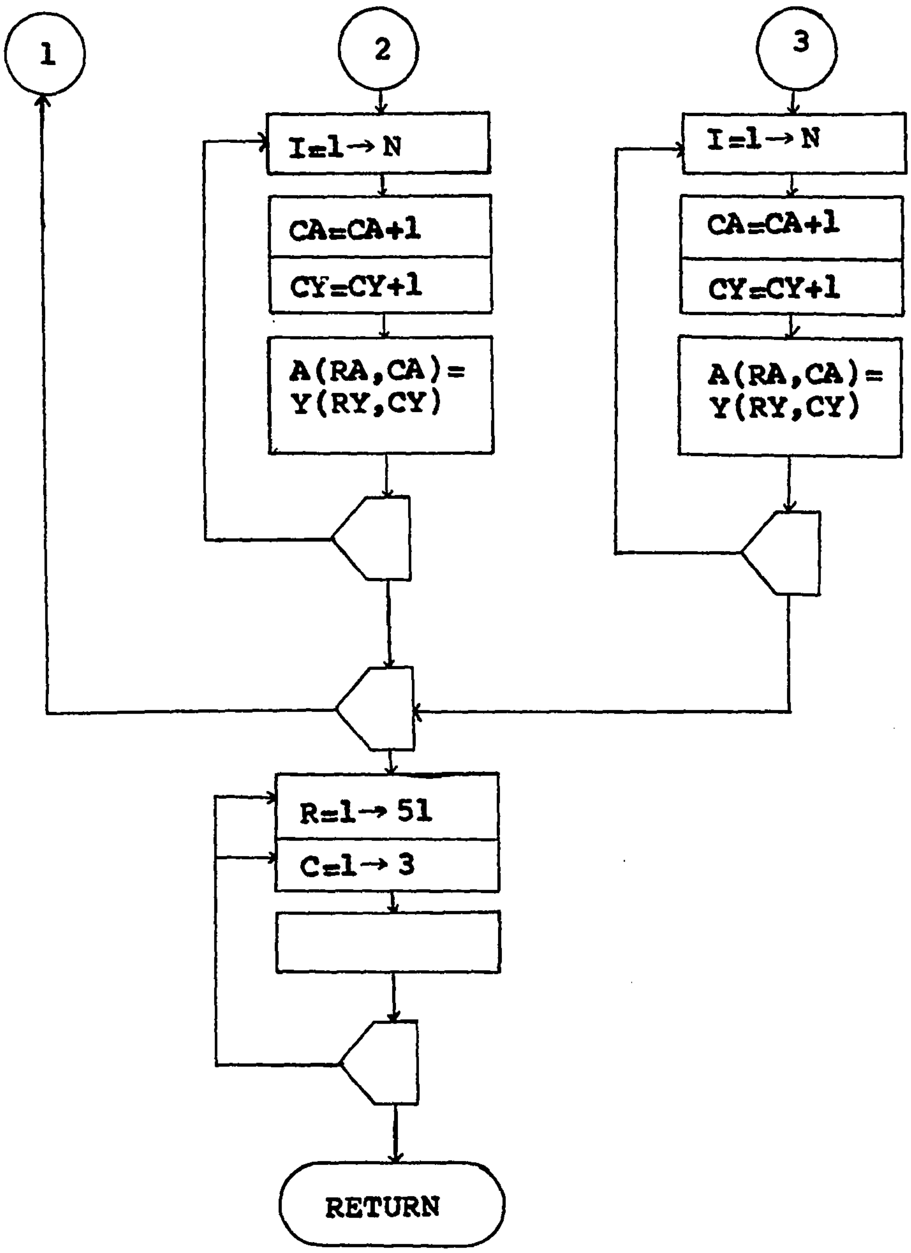


Figure 13. (continued)

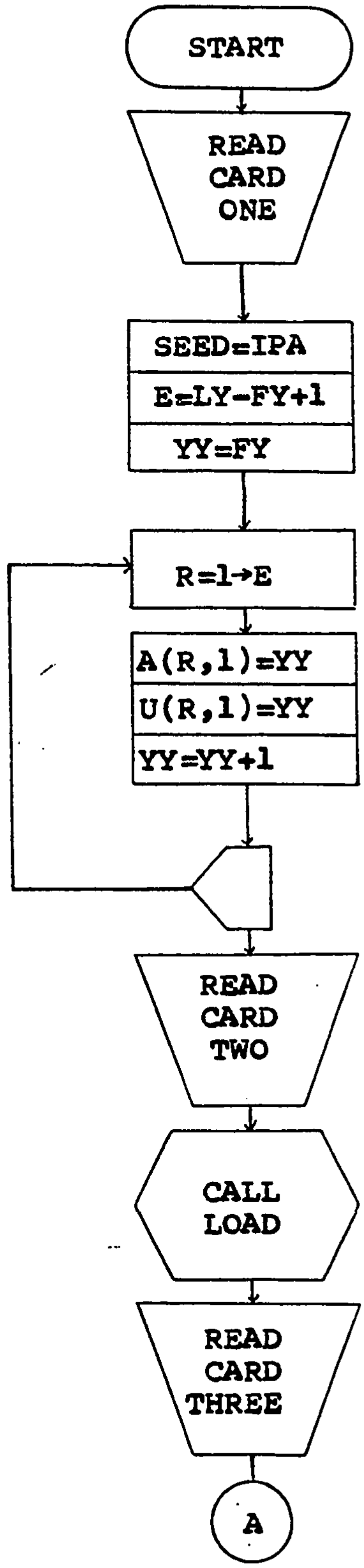


Figure 14. INPUT Subroutine

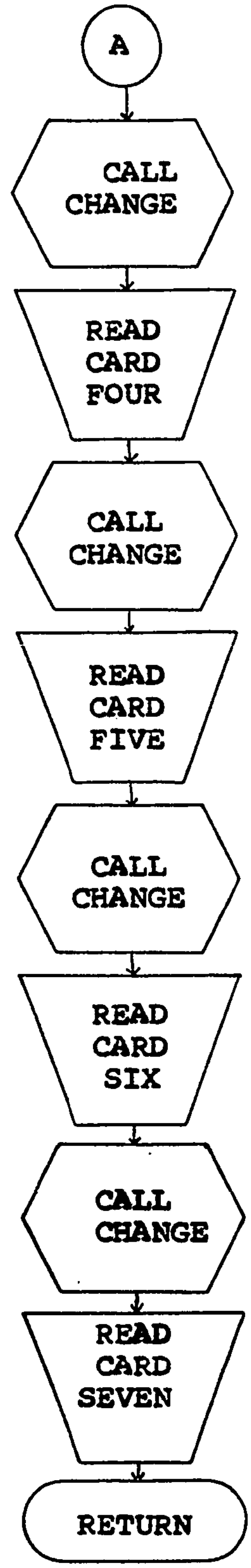
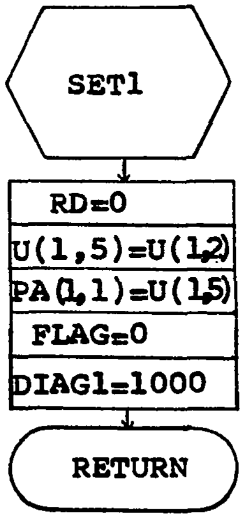


Figure 14. (continued)



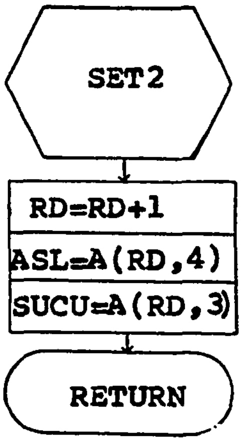


Figure 15. SET Subroutines

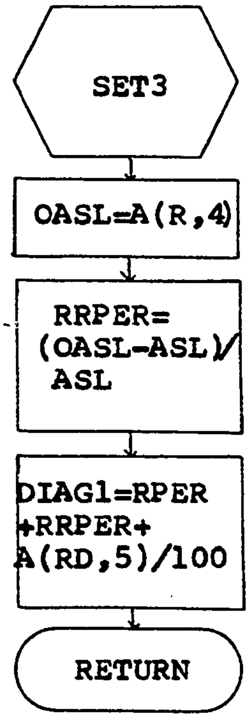


Figure 15. (continued)

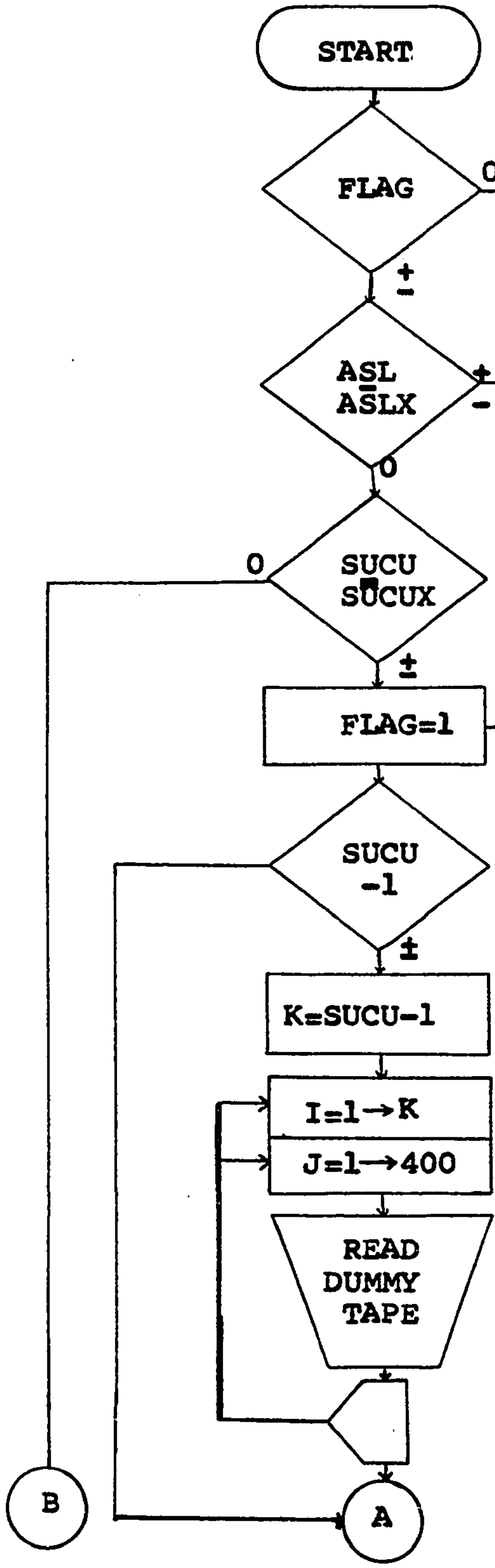


Figure 16. DMATRX Subroutine

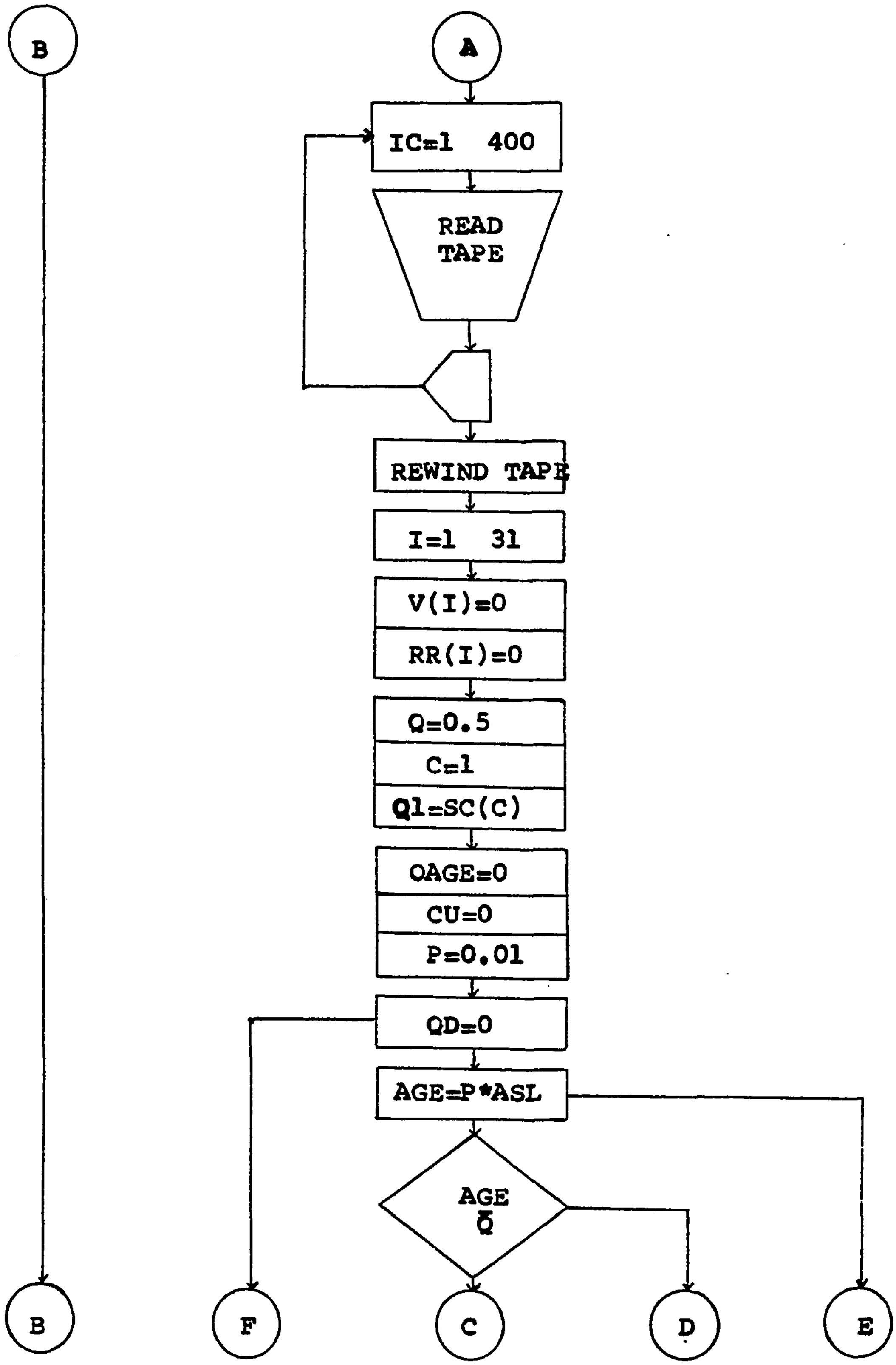


Figure 16. (continued)

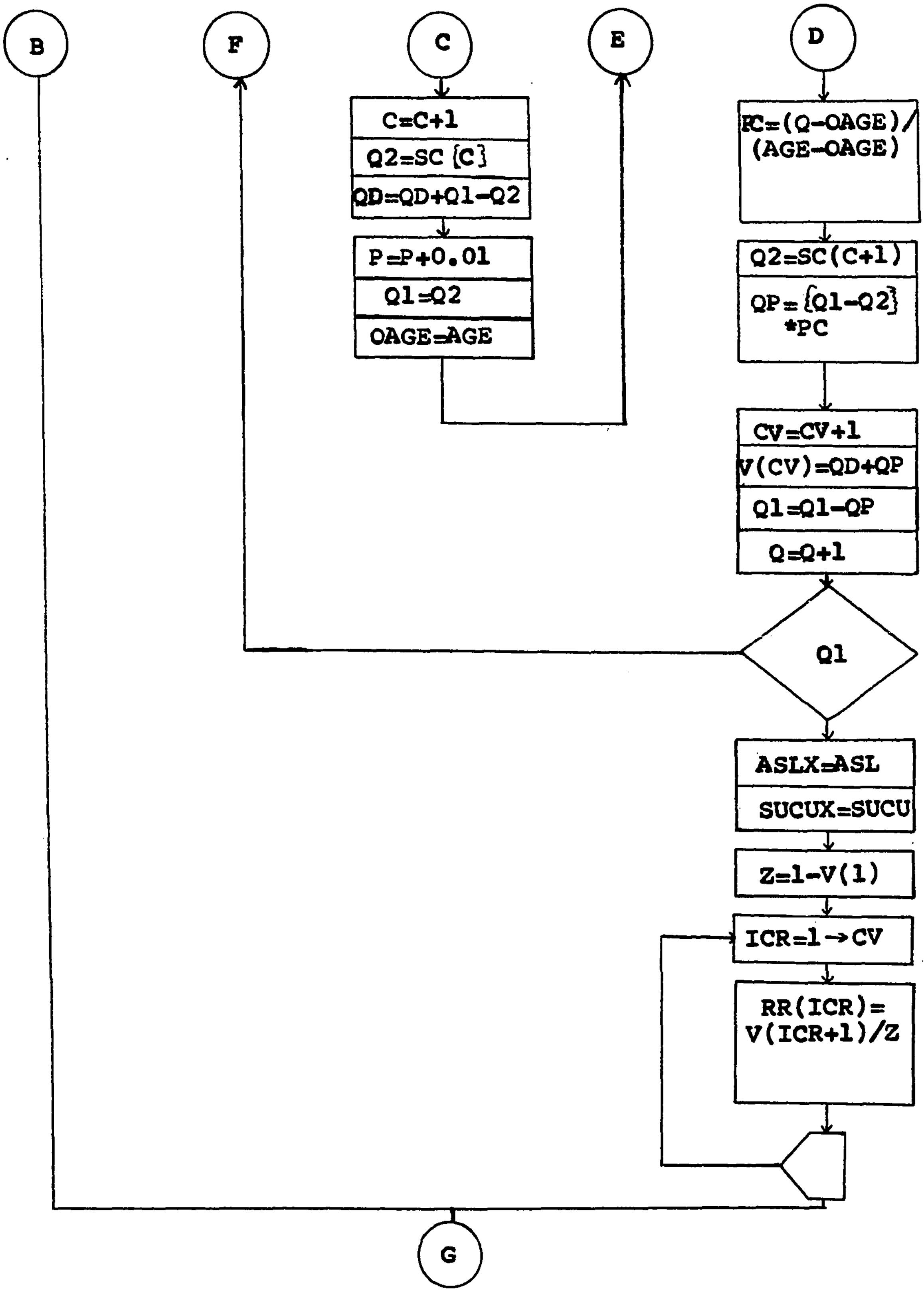


Figure 16. (continued)

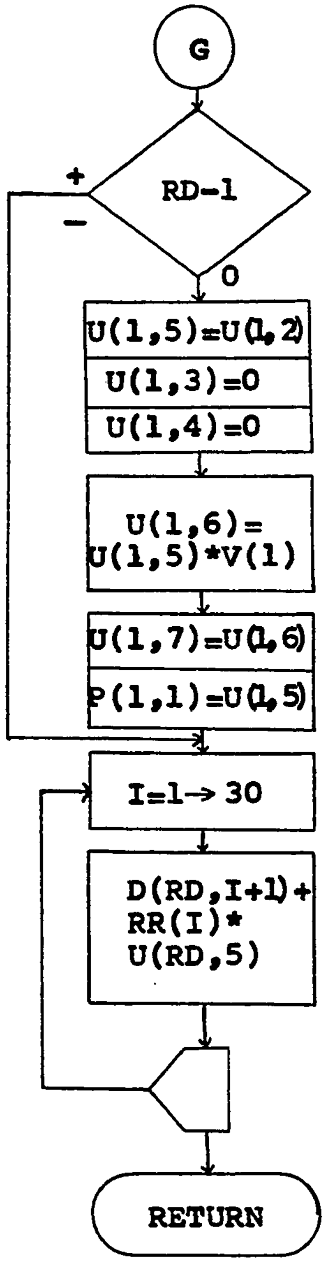


Figure 16. (continued)

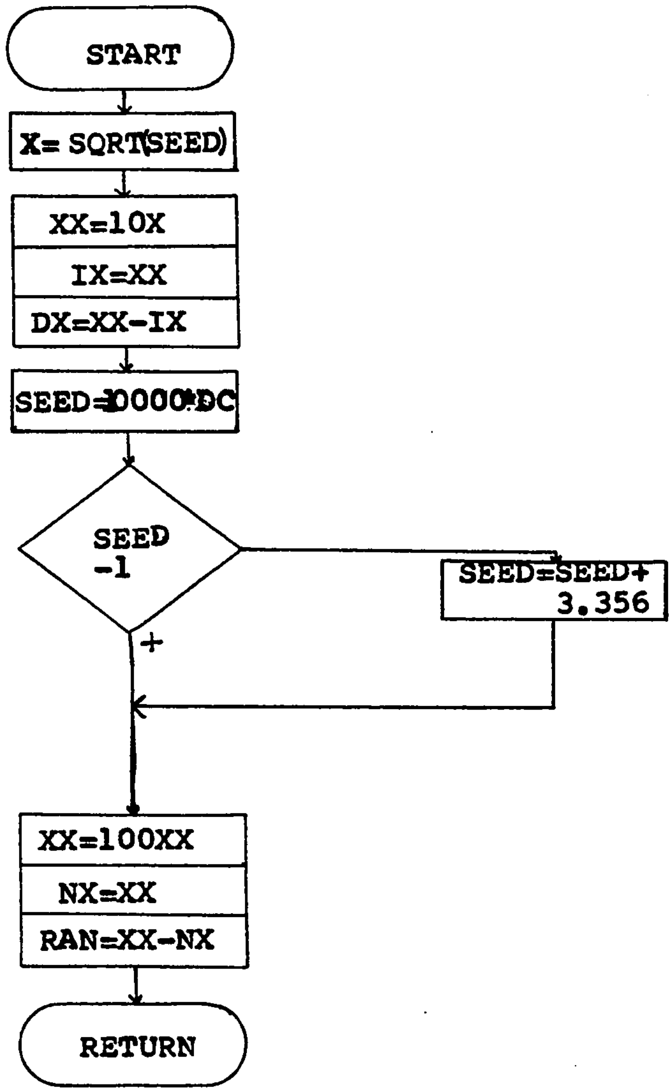


Figure 17. RANDOM Subroutine

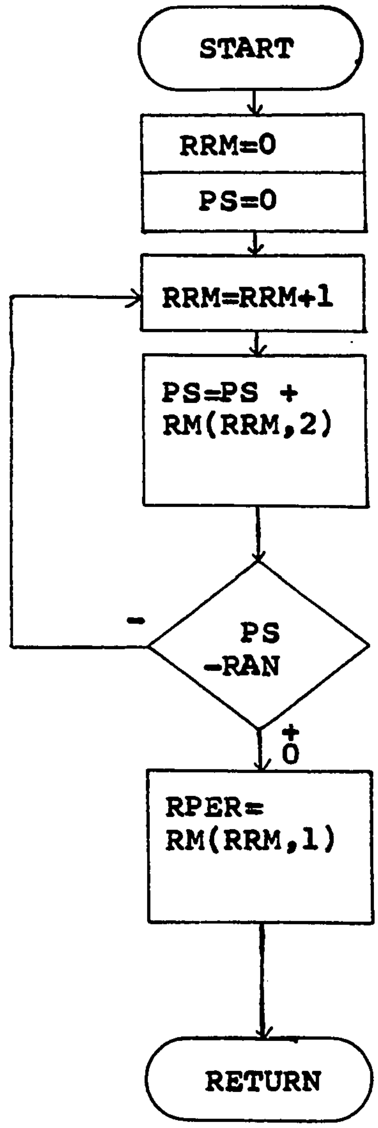


Figure 18. RANPER Subroutine

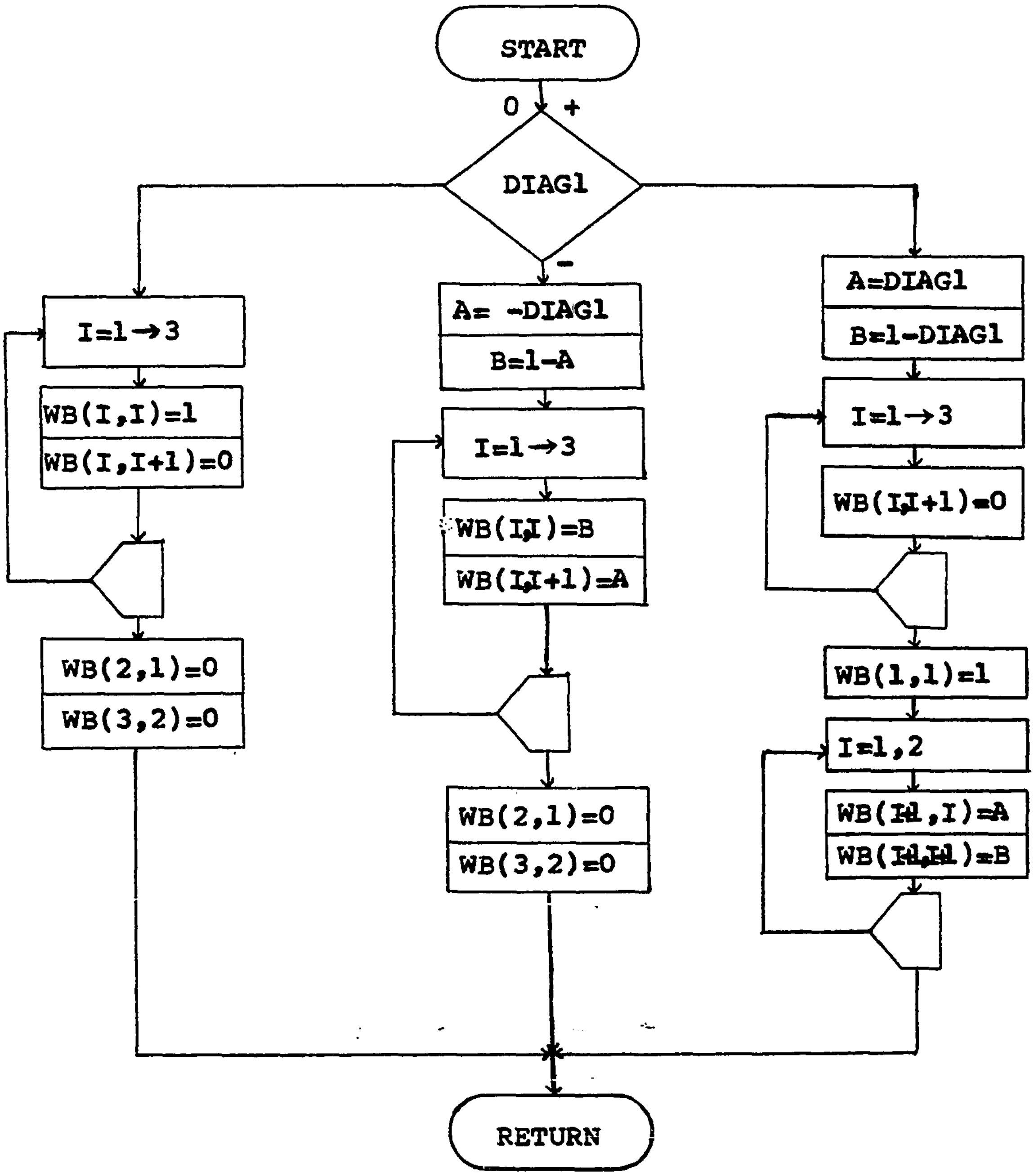


Figure 19. PMATRX Subroutine

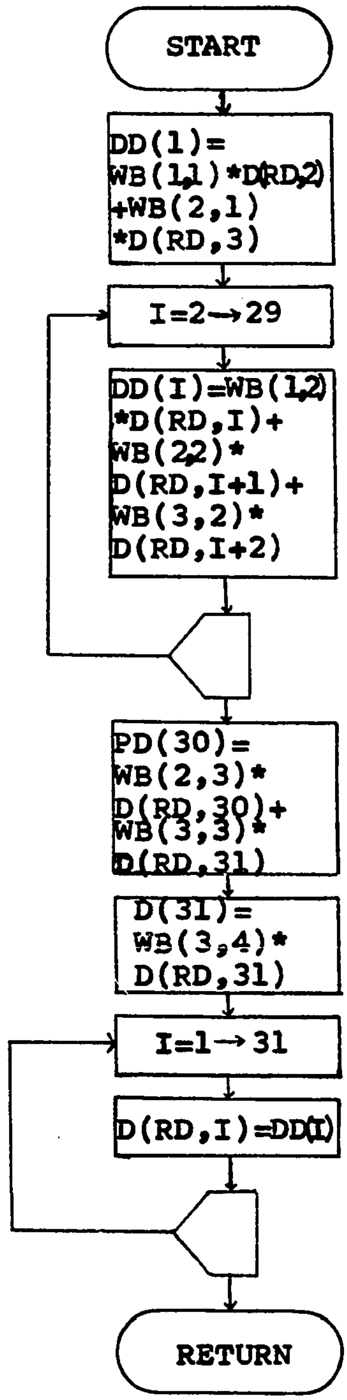


Figure 20. DMULTP Subroutine

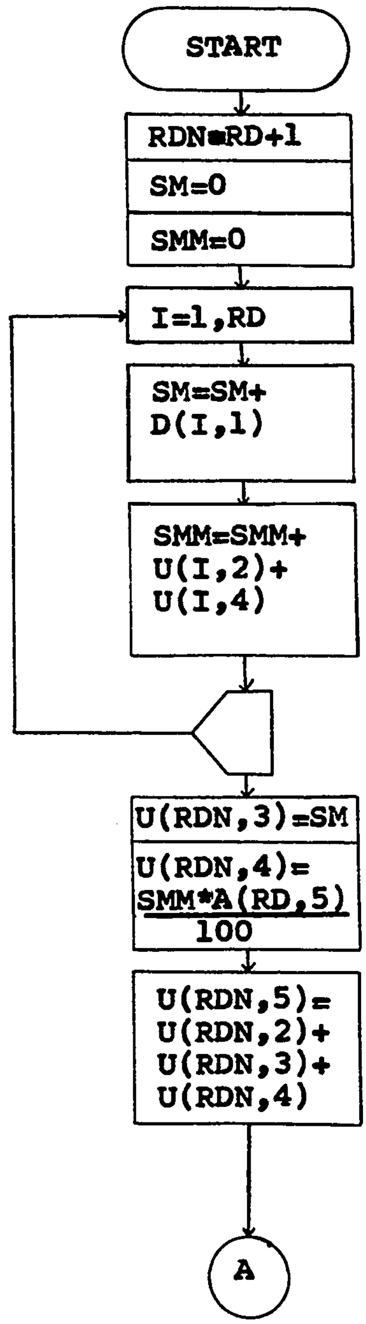


Figure 21, RENWL Subroutine

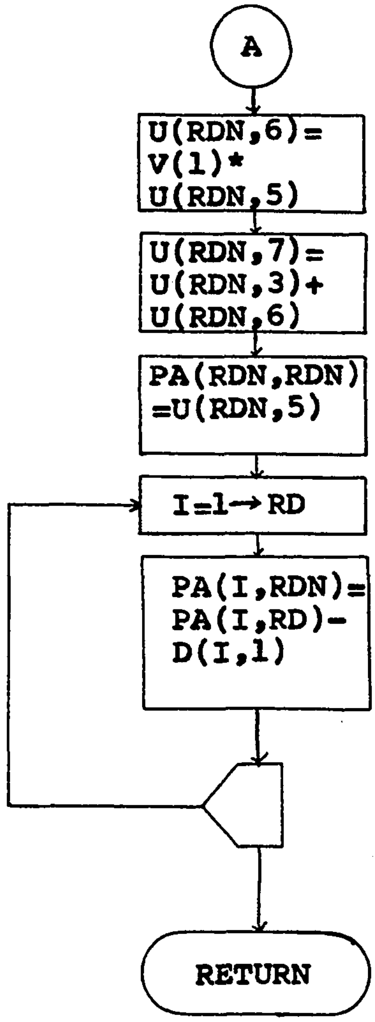


Figure 21. (continued)

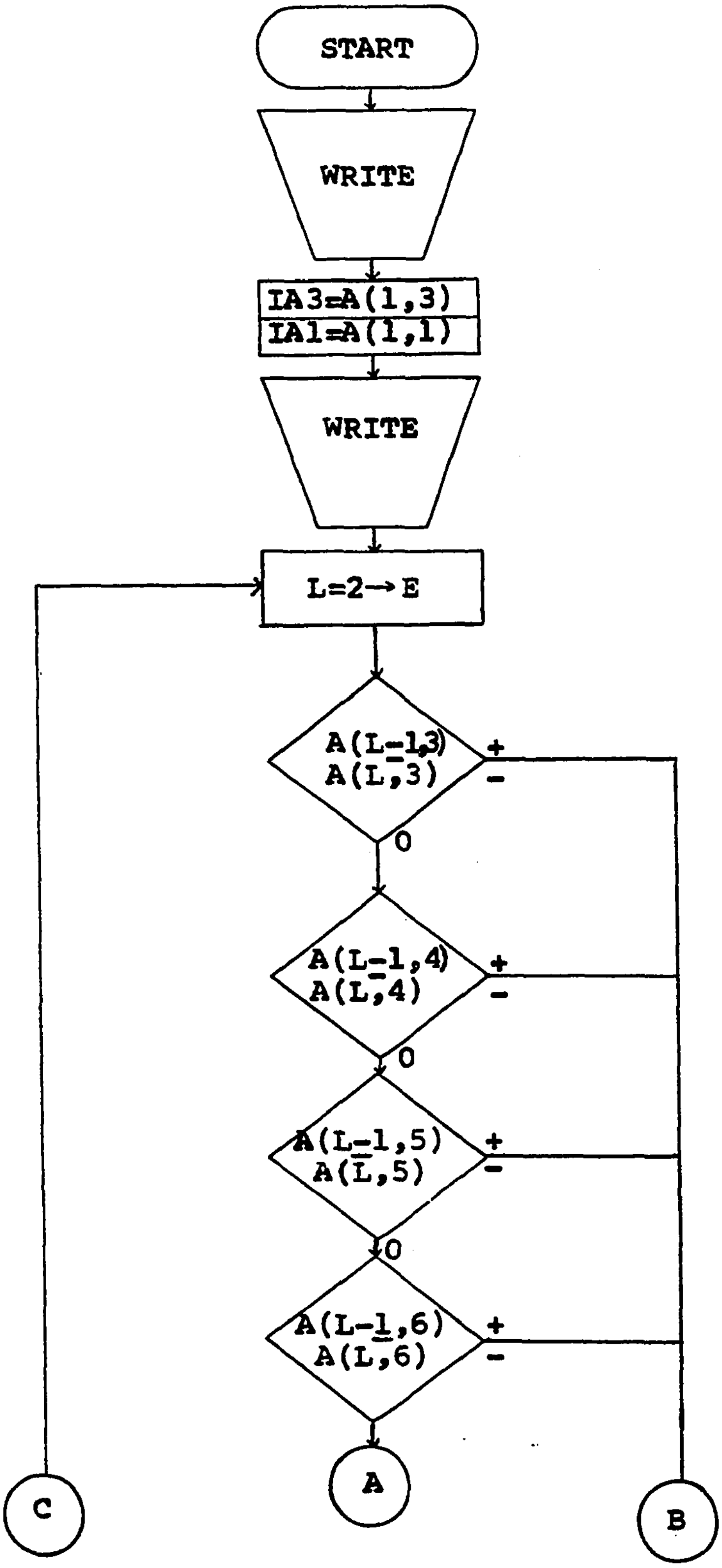


Figure 22. PAGE Subroutines

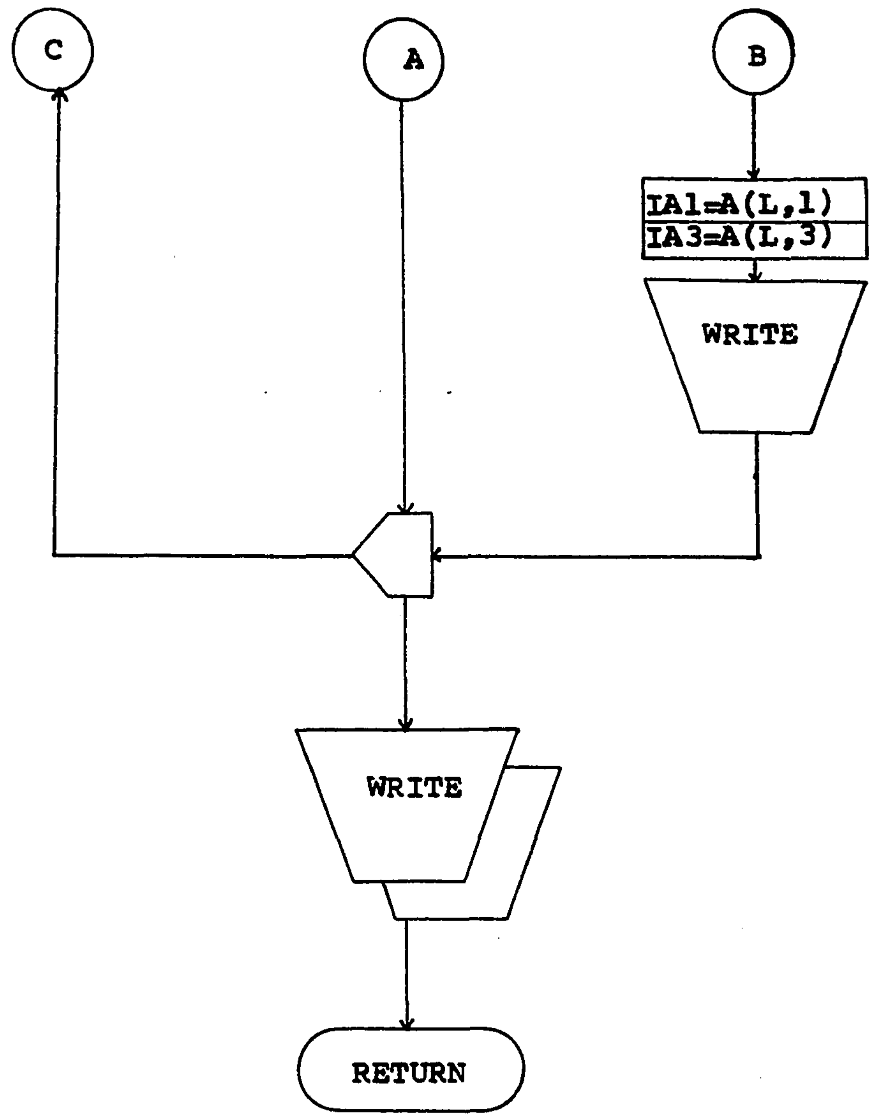


Figure 22. (continued)

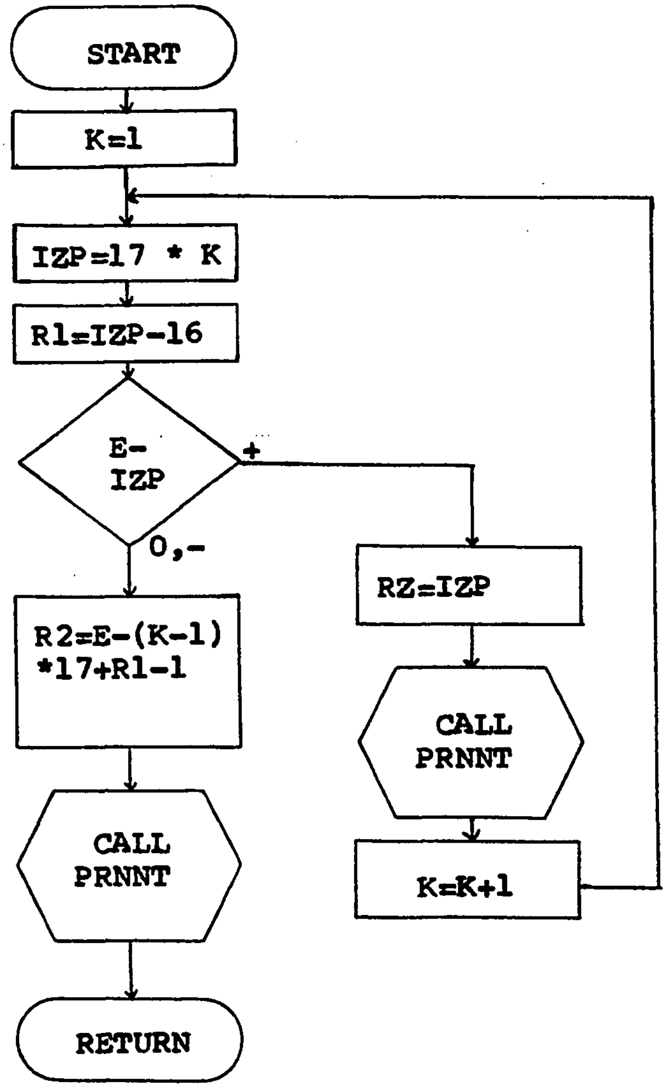


Figure 22. (continued)

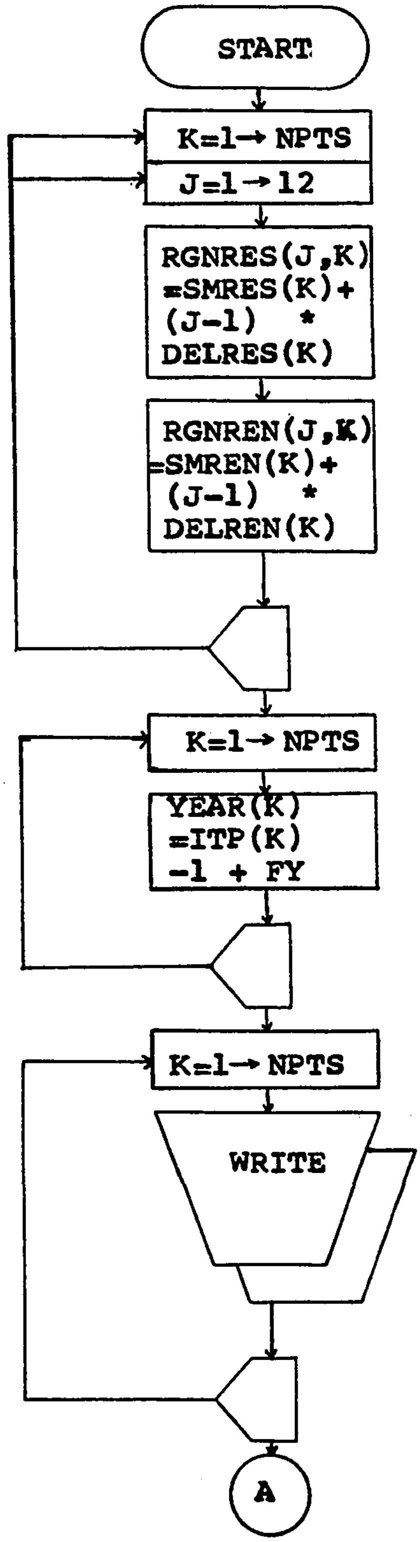


Figure 22. (continued)

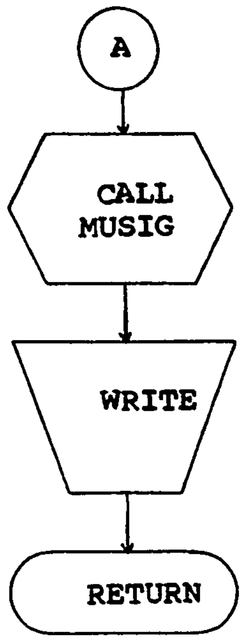


Figure 22. (continued)

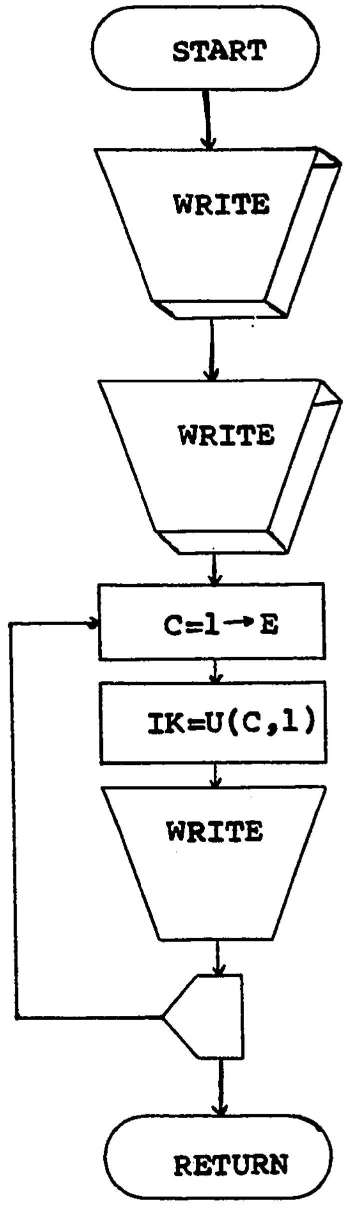


Figure 23. PRNNT Subroutine

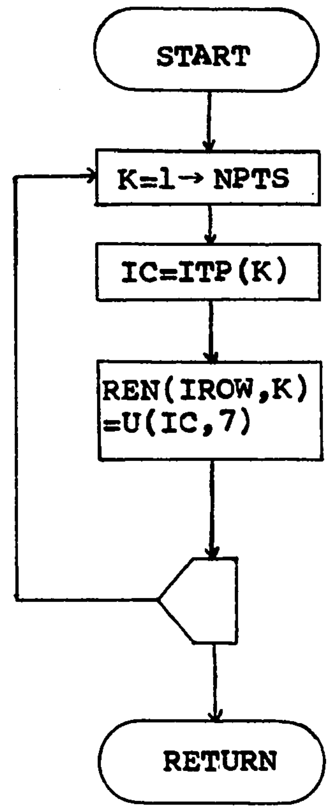


Figure 24. HISTO Subroutines

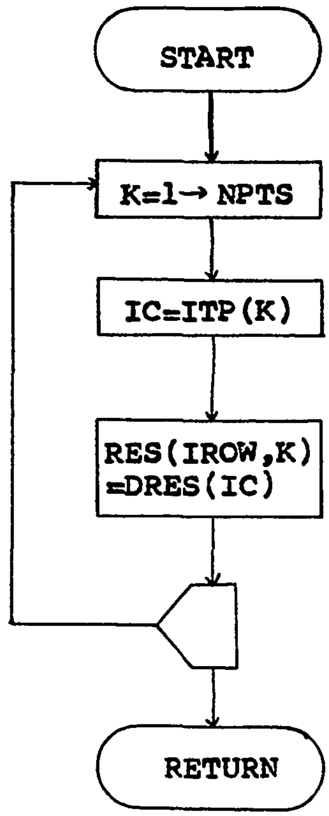


Figure 24. (continued)

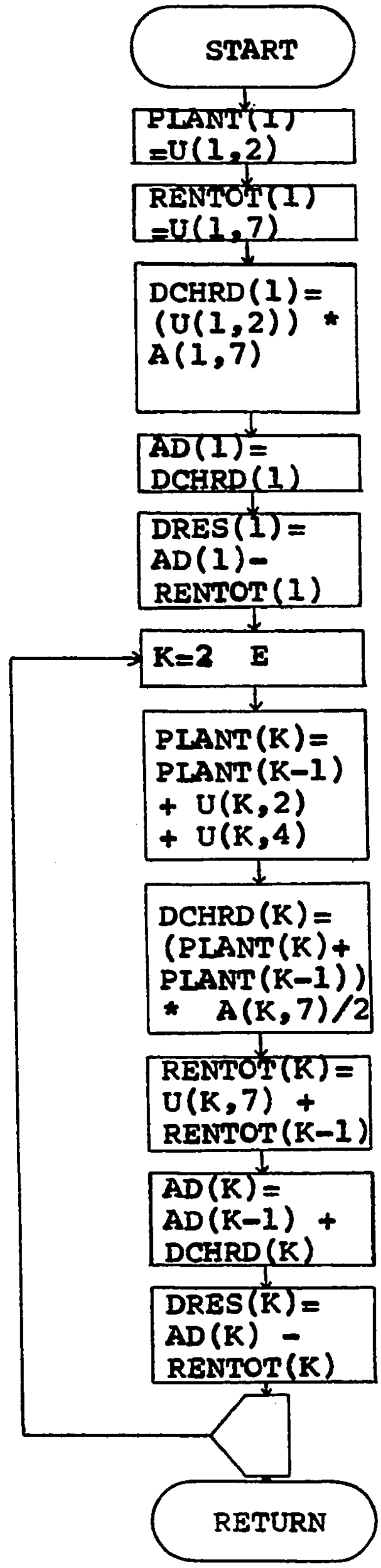


Figure 25. DEPRES Subroutine

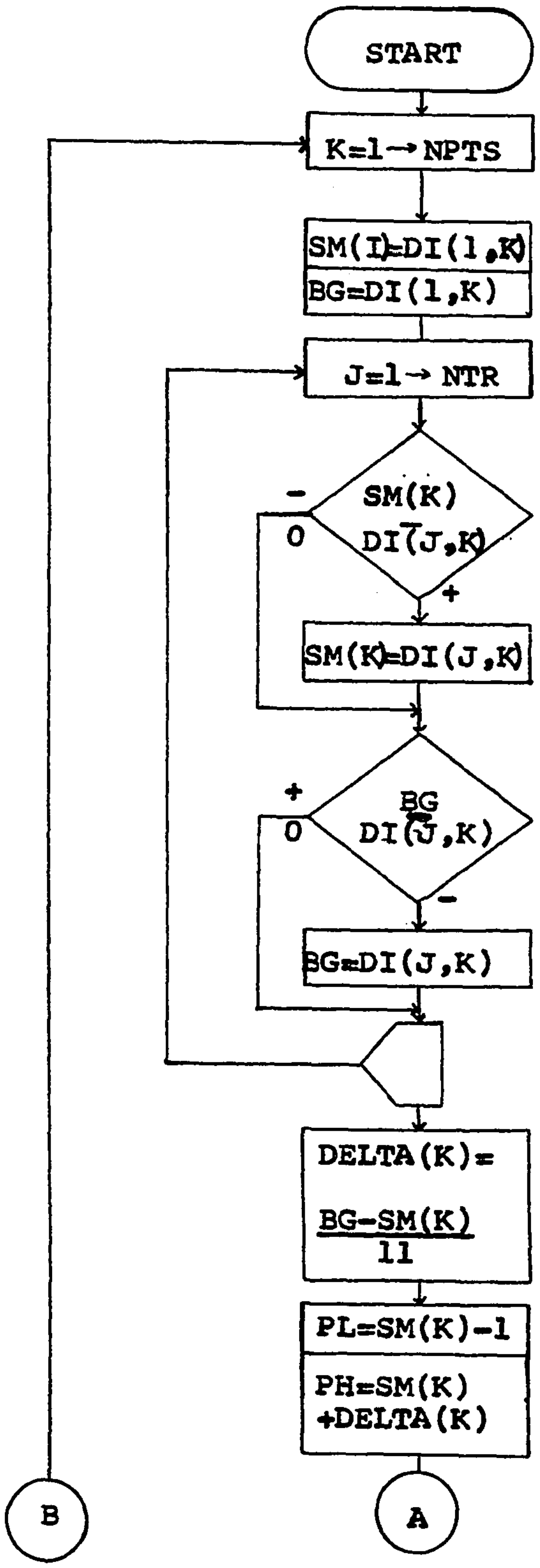


Figure 26. HISAGN Subroutine

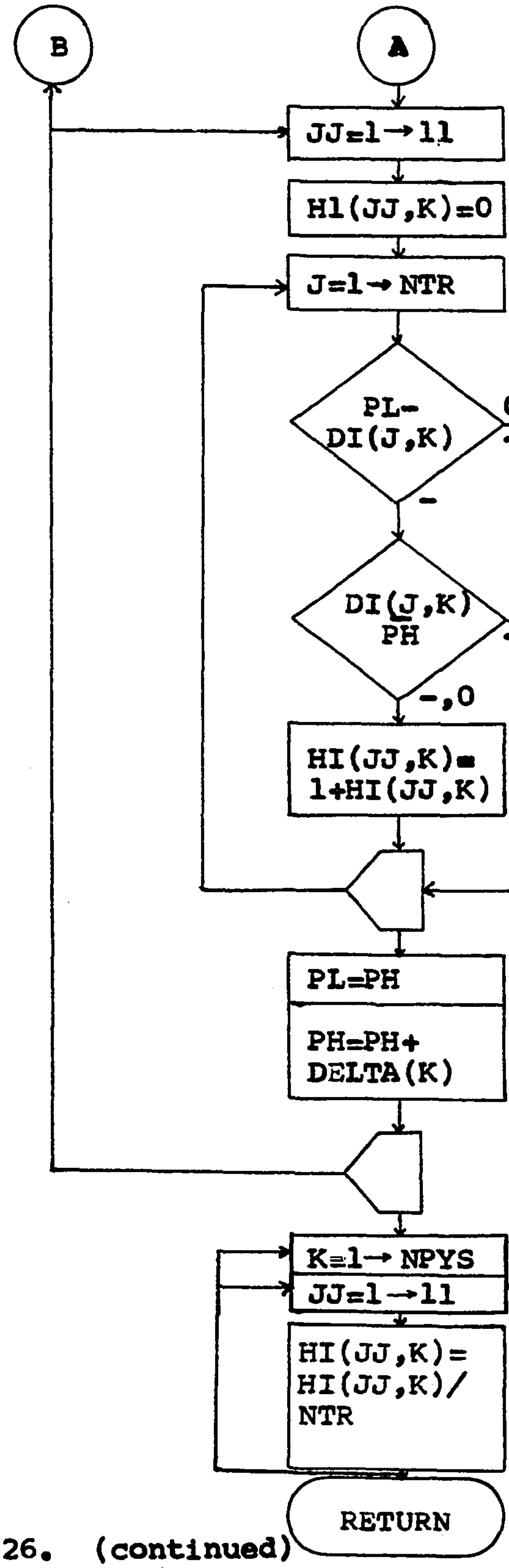


Figure 26. (continued)

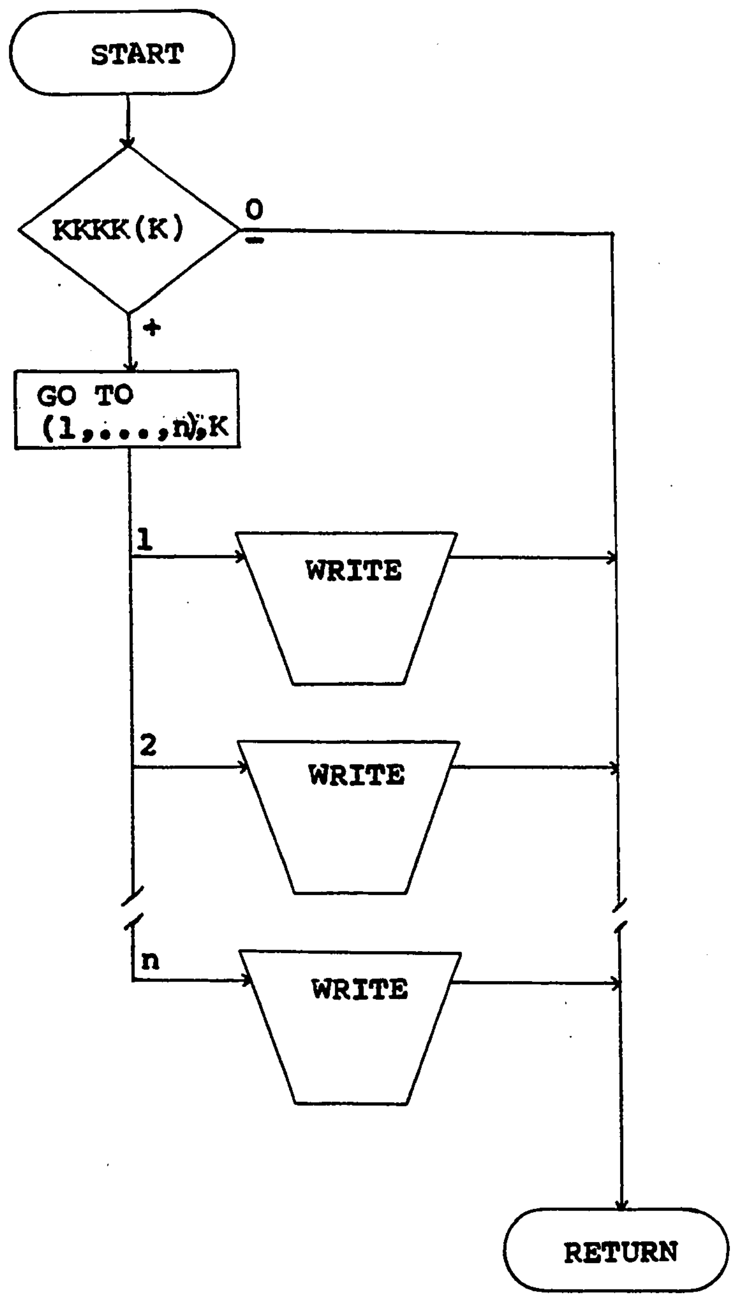


Figure 27. BUG Subroutine

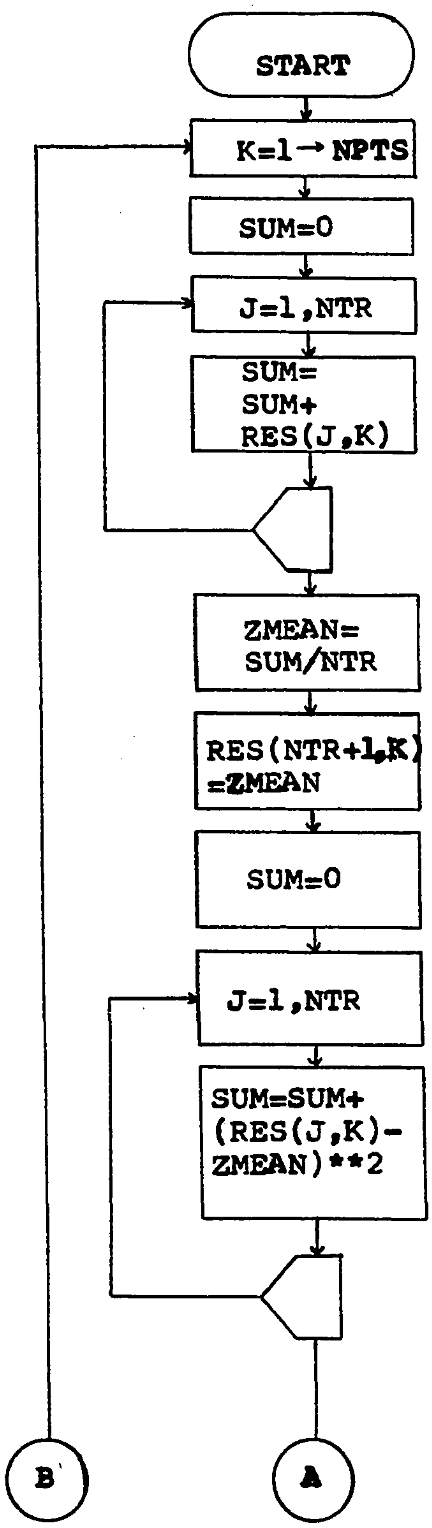


Figure 28. MUSIG Subroutine

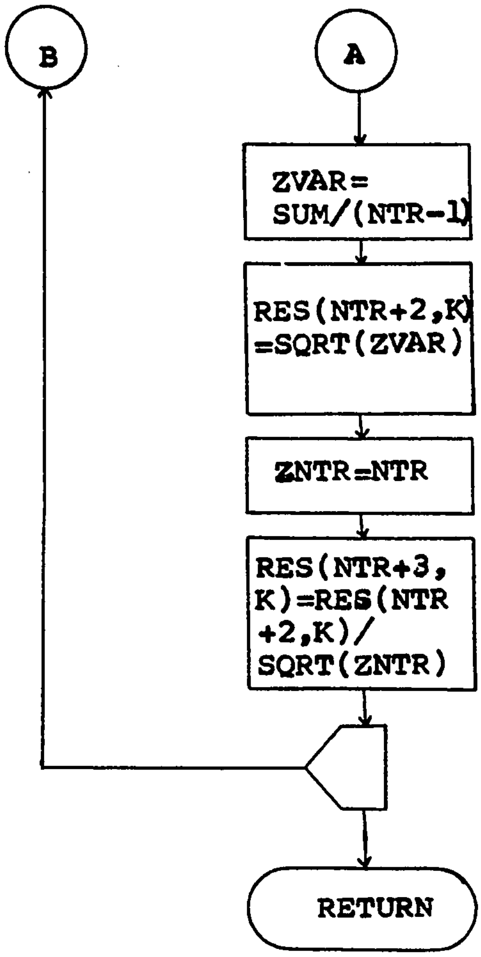


Figure 28. (continued)

Section C: Main Program and Subroutines

MAIN Program

The first row data cards read by the program will respectively denote the information the debug subroutine BUG will supply, and the number of accounts NR which will be simulated.

The first loop cycles NR number of times, after which the ZERO subroutine is called to set all arrays to zero. Next, the INPUT subroutine is called, which reads the data from six cards whose information specifies the parameters of the first of NR accounts. In order to utilize the account specifications efficiently the CHANGE and LOAD subroutines place the information on a yearly basis in the A array.

The second loop cycles NTR times where NTR equals the number of simulations of the account which have been specified. The SET1 subroutine immediately following the beginning of the second loop initializes parameters. The SET2 and SET3 subroutines accomplish the same function though they immediately follow the third and fourth loops respectively.

The third loop cycles E times where E is a number of years which the account will run. Following SET2 subroutine the DMATRX subroutine distributes the property in array D to be placed in service that year according to the property life distribution. Each year the account is in operation is represented by one vector of the two dimensional array D array. At this point the random numbers relating to the variation in property flowthrough are generated for the P array. The random number is generated by the RANDOM subroutine and the percentage variation is calculated by the RANPER subroutine.

The fourth loop cycles RD number of times where RD is the year of operation being worked on in the third loop. If it is the first time through for the routine, RD=1 or the percentage variation in flowthrough is unequal to the previous year, DIAG1 unequal DIAG2, then the PMATRX subroutine is called. The PMATRX subroutine sets up the diagonal elements of the P array by placing them in the more compact yet equivalent WB array. In either case the DMULTP subroutine is next and it multiplies the P array times the D array to make a new D array which represents the impact of one year passing on RD accounts.

Following the exit from the fourth loop the RENWL subroutine places the number of placements, renewals, growth, renewal adjustments, and retirement information in the U matrix. It also updates the property account array PA.

After the third loop is completed the DEPRES subroutine calculates depreciation charges, accrued depreciation, accrued renewals, plant in service, and depreciation reserve. If IOPA is one, the computer calls PAGE1 subroutine which prints a summary of the account specifications. The PAGE2 subroutine is next called and in conjunction with the PRNNT subroutine prints out the property account array PA, the U array, and the information calculated in the DEPRES subroutine. If IOPA is zero, this is one run of NTR runs, and IOH should be set to one. When IOH equals one, the depreciation reserve and renewal information at various points in time are collected for subsequent arrangement in a histogram. Then HISTO1 and HISTO2 are called for collecting this information about the depreciation reserve and renewals respectively.

The end of the second loop completes the number of trials selected from one account. Immediately if IOH equals one, the information collected by the HISTO1 and HISTO2 subroutines is arranged in a histogram by the HISAGN subroutine. A summary of account specification is printed by calling the PAGE1 subroutine. The histograms of renewals and depreciation reserve are printed by calling the PAGE3 subroutine. The end of loop one signals that all accounts have been processed by stopping the program.

INPUT Subroutine

The property account is described by six cards hereafter referred to as the set. Card one of the set contains seven items of information. Item number one is the number of years the account will run. Variable name FY is the first year, and LY is the last year. The second item of information is the property account number denoted by IPA. Item number three is the number of trials to be run which is listed under the variable name NTR. The fourth item tells whether the property account shall be printed. If IOPA is one, the account will be printed. If IOPA is zero the account will not be printed. If IOH is one, the histogram for renewals and depreciation reserve of an account is printed, whereas if IOH is zero, no histograms will be printed. The sixth item of information is the number of observation points used in the property account. This is represented as variable NTR which must be less than ten. The last bit of information is the specific years from the first year which are to be observed. These are put in the ITP array. If, for example, FY equals 1961 and ITP(k) equals 5 then the k observation is in 1965.

Card two of the set denotes the times at which major property placements occur. The variable N (N less than 50) represents the number of placements, and the year Y(2,k) that amount which was placed in service where k=1,2,...,N.

Card three of the set describes the life distribution of the property which is placed in serivce. The number of changes in the property distribution is N. The year that the property is experiencing a particular life pattern is Y(1,k) where k=1,2,..,N. The life patterns used are the

Iowa Survivor Curves. The curve numbers and average service life are represented by Y(2,k) and Y(3,k) respectively.

Iowa Survivor Curve	#
Square Curve	1
SC(.5)	2
SO	3
SO(.5)	4
S(1)	5
S(1.5)	6
S(2)	7
S(3)	8
S(4)	9
S(5)	10
S(6)	11
SQ	12
L(0)	13
L(.5)	14
L(2)	15
L(1.5)	16
L(2)	17
L(3)	18
L(4)	19
L(5)	20
R(0.5)	21
R(1)	22
R(1.5)	23
R(2)	24
R(2.5)	25
R(3)	26
R(4)	27
R(5)	28
O(2)	29
O(3)	30
O(4)	31

Card four of the set depicts the growth patterns of the property account. Variable N represents the number of growth specifications over the life of the account. Array Y(1,k) denotes the year the k growth pattern is present and

Y(2,k) holds the growth factor where k=1,2,..,N.

Card five of the set sets the trend retirements. The number of trends is set to the variable N. The year the trends are in effect is in the array Y(1,k). The trend percentage is in array Y(2,k) where k=1,2,..,N.

Card six of the set determines the depreciation rate. The number of rate changes is set equal to the variable N. The year the rates are in effect is in the array Y(1,k). The rate itself is in array Y(2,k) where k=1,2,...,N.

Card seven is the randomization distribution of the property flowthrough. N represents the number of possible variations in which N must be less than twenty. The percentage probability of occurrence is RM(1,k) where k=1,2,...,N. The respective occurrence is found in RM(2,k). The N is never changed and is used once again in subroutine PAGE1.

LOAD and CHANGE Subroutines

All the account specifications are read into the Y array. Using the LOAD and CHANGE subroutines this information is transferred to the A array. The following illustration demonstrates the use of these subroutines.

Y Array 2 card Placements		Y Array 3 card Mortality Law			Y Array 4 card Growth		Y Array 5 card Trend		Y Array 6 card Depreciation Rate	
Year	$000	Year	#	ASL	Year	%	Year	%	Year	Rate
1961	10	1961	8	5	1961	0	1961	0	1961	.200
1961	15	1664	18	7	1963	-5	1964	3	1965	.144

The above data which is assimulated into the A array is demonstrated below.

Year	Placement	Mortality Law	ASL	Growth	Trend	Rate
1961	$10,000	8	5	0%	0%	.200
1962		8	5	0%	0%	.200
1963		8	5	-5%	0%	.200
1964		18	7	-5%	3%	.200
1965	$15,000	18	7	-5%	3%	.144

The LOAD subroutine puts the placement data in the A array. The CHANGE subroutine places the rest of the data. In order to place the information in the correct columns, the variable FCA denotes the column of A in which the information shall be placed. For example, when the fourth card containing the growth information is read and the CHANGE subroutine is called, FCA is set to five, whereas when the third card was read and the CHANGE subroutine called, FCA was set to three.

The SET1, SET2, and SET3 Subroutines

Before entry is made into loops two, three, and four of the main program, certain variables must be initialized. These initializations are accomplished by the SET subroutines.

DMATRX Subroutine

The DMARTX subroutine distributes the property placed in service in the appropriate vector of the D array. The life distribution is calculated and found on the RR vector. U(RD,5) represents the amount of property to be placed in the RD year.

The DMATRX subroutine first establishes if there has been any change in the RR vector or if the DMATRX has ever been called. If not, then D(RD,k) is calculated immediately. If one of the above conditions exists the appropriate portion of the tape is read so as to put the Iowa Survivor Curve into the SC vector. The average service life of this initial curve is one hundred years. Using linear interpolation, the percent surviving each year for the desired average service life is calculated and placed in the vector representing f(k). The program constrains the life of any one property unit to a maximum of thirty years. In order to adjust for retirements during the first half year of a placement, the RR vector representing g(k) is calculated to represent subsequent retirements. The k value of RR is RR(k) = V(k+1)/z where z = 1-V(1). In the last step U(RD,5) is distributed in the RR vector of the D array as D(RD,k+1)=RR(k)*U(RD,5). This leaves D(RD,1)=0 which implies no immediate retirements. The variable RR is the year of simulation for the property account.

RANDOM AND RANPER Subroutines

These two subroutines are used together and will also be discussed as such. The RANDOM subroutine generates a random number to be used in determining the percentage variation from the RM distribution. The seed is the property account number(SEED=IPA). The random number RAN is the seventh and eighth digits of the square root of the previous seed. If the new SEED is less than or equal to one it is incremented by 3.356.

The RANPER subroutine takes the random variable RAN and finds the associated percent deviation which is called RANPER. This technique is commonly referred to as Monte Carlo simulation.

PMATRX subroutine

This routine sets up the P matrix, which is now referred to as WB, so that the proper flowthrough of property in the D array may be calculated. The WB matrix is a 3x4 which contains all the needed information so the above mentioned multiplication can take place. Let a be an element of WB then the following matrix represents all of the elements of WB.

$$\begin{matrix} a(1,1) & a(1,2) & a(1,3) & a(1,4) \\ a(2,1) & a(2,2) & a(2,3) & a(2,4) \\ a(3,1) & a(3,2) & a(3,3) & a(3,4) \end{matrix}$$

If the flow through of property in D is faster that expected the a(1,1)=1, a(2,1)=a(3,2)=k the percent increase, a(2,2)=a(3,3)=1-k and all other elements equal zero. On the other hand if there is a delay in property retirements then a(1,2)=a(2,3)=a(3,4)=k the percentage delay, a(i,i)=1-k, and all other elements equal zero. If there is no change in flowthrough then in either case k=0.

DMULTP Subroutine

The DMULTP subroutine multiplies the two matrices P and D. This routine multiplies one vector of D at a time by P. In order not to destroy the information in D an intermediate vector DD is used to hold the information. DD(1) is a function of the first column of WB. DD(2) through DD(29) are functions of the second column of WB. DD(30) and DD(31) are functions of the third and fourth columns of WB respectively. The final calculation is D(RD,k)=DD(k) which transfers the desired information to D.

RENWL Subroutine

After one through RD vectors of the D array have been operated on, the RENWL subroutine sets up the information to be used in the next year's operation. The number of retirements which took place are labeled U(RDN,3) where RDN=RD+1. The amount of growth is calculated in U(RDN,4), and U(RDN,5) is the total property to be placed in service, a summation of U(RDN,k) where k=2,3,4. The half year adjustment is calculated and placed in U(RDN,6) and total retirements are U(RDN,7)=U(RDN,3)+U(RDN,5). All previous property accounts are updated by subtracting the retirements from the survivors in each account by PA(k,RDN)=PA(k,RD)-D(k,1) where k=1,2,,...,RD.

PAGE1, PAGE2, PAGE3, and PRNNT Subroutines

PAGE1 is a subroutine which prints a summary of the property account specifications listed in the six sets of data cards. This routine is called before either PAGE2 or PAGE3 is required.

The subroutine PAGE2 determines the variables R1 and R2 used in the PRNNT routine. The PRNNT subroutine writes out the property account PA, the information in the U array, and lists the values calculated in the DEPRES subroutine.

Only seventeen years of information may be printed per page. Thus the use of variable R1 and R2 is to control the amount of output per page. The subroutine PAGE3 prints the histograms which have been calculated by the HISAGN subroutine.

HISTO1 and HISTO2 Subroutines

HISTO1 takes the appropriate information from the retirement U(k,7) vector as k as dictated by the ITP vector, and places it in the REN array. HISTO2 does the same for the depreciation reserve vector DEPRES(k) except these facts are stored in the RES array. The columns of RES and REN represent the particular years of interest and the rows are the number of simulations being observed.

DEPRES Subroutine

This routine calculates the depreciation charge DCHRD(k) in the k year using the average life procedure. Other information calculated is the depreciation reserve DEPRES(k), the accrued depreciation AD(k), the accumulated renewals RENTOT(k), and the property in service PLANT(k).

HISAGN Subroutine

The HISAGN arranges the information accumulated in the RES and REN arrays into a histogram. The information is divided into eleven groups. The smallest value SM(k) of each test period k is retained as well as the range of each group DELTA(k). Once the range is established (PL,PH) for the histogram HI(i,k) where i=1,2,...,11 and k=1,2,...,NPTS the DI array is search for a number in that range. If one is found it is noted in the HI(i,k) array. When all numbers of DI have be noted in whatever range they belong, HI(i,k) is divided by NTR to place the histogram on a decimal basis.

MUSIG Subroutine

This routine calculates the mean and variance of the depreciation reserve. It also calulates the standard deviation of the distribution of means. These facts are placed in the NTR+1, NTR+2, and NTR+3 positions respectively of the RES array in the appropriate year. MUSIG is called from with in the PAGE3 subroutine and returns the aforementioned information to be printed.

BUG Subroutine

This was a major debug routine. Programming experience dictates it to be helpful if a debug routine is written right into the main program and subroutines. It was hoped such a routine would help simplify the debugging and testing process. Anytine output from any point was desired the point was labeled k, and CALL BUG(k) was written. K was a fixed number and referred to a particular write statement in the BUG subroutine. Before the subroutine printed any information the array KKKK(k) was checked to ascertain whether it was zero or one. If zero, no output was printed, or if one, the write statement was reached, and the desired information was printed. In this manner the CALL BUG(k) statement could be placed and left in the program subject to use only if the very first card read contained a one in the k column. This routine kept the program legible in early debug and test stages. Accordingly it was left in for future users.

section D: Major Arrays and Variables

This section describes the purpose of major arrays and variables which are listed alphabetically.

1. A(51,7)--Property specifications are held in this array. Column one holds the years of the property account in consecutive order. The dollar of property placed, Iowa Survivor Curve number, average service life, growth factor, and trend factor are found in columns two through six respectively.
2. AD(51)--The accumulated depreciation charges for each year are stated in the AD array.
3. ASL--The average service life of the year of the property account being simulated.
4. ASLX--The average service life of the previous year.
5. CKY--The label for the actual year.
6. D(51,31)--Each new vector holds the distribution of life remaining for property placed in that year. The element D(r,c) represents the amount of property with c-1 years of life remaining which was placed in service in the r year.
7. DCHARD(53)--This array holds the depreciation charges. The information is printed by the PAGE2 subroutine.
8. DD(31)--When each row vector of the D array is updated the information is temporarily stored in this array and

eventually transferred to D.

9. DELREN(10)--This array is equivalent to DELTA(10) found in the HISAGN subroutine. It holds the renewal histograms ranges for each of the ten observations points. The information is used in the PAGE3 subroutine which prints the histogram.

10. DELRES(10)--Used in the same manner as DELREN(10) except it stores range information concerning the depreciation reserve.

11. DELTA(10)--The array is used in the HISAGN subroutine and is equivalent to DELRES(10) or DELREN(10).

12. DI(100,10)--This array is used in the HISAGN subroutine and is equivalent to REN(100,10) or RES(100,10).

13. DIAG1--The diagonal information of the P matrix is held and is used to set up the compact WB(3,4) matrix.

14. DIAG2--The variable retains DIAG1 information used previously. If FLAG equals one and DIAG1 equals DIAG2 then the PMATRIX subroutine is not used for that year.

15. DRES(50)--The depreciation reserve per year is stored and the information is printed by the PAGE2 subroutine.

16. E--A variable calculated form FY-LY+1 which represents the duration in years of the property account being simulated.

17. FCA--A column counter preset by the calling of subroutine CHANGE. It denotes the column of the A array that the

information from the Y array will be placed.

18. FLAG--If FLAG is zero, the complete DMATRX subroutine is utilized, otherwise it is optional subject to use if the Iowa Survivor Curve changes.

19. FY--The variable represents the first year of the property account.

20. HI(11,10)--This array is used in the HISAGN subroutine and is equivalent to RENH(11,10) or RESH(11,10).

21. IOH--This variable controls whether HISTO1, HISTO2, and HISAGN subroutines are entered and whether the PAGE3 subroutine is printed.

22. IOPA--This variable controls whether PAGE1 subroutine is printed.

23. IPA--The property account number is held under this variable name.

24. IROW--IROW is the counter of the number of simulations to be made on each account.

25. ITP(10)--The ITP array holds the position in relation to FY at which the variation in renewals and depreciation reserve is to be observed. A maximum of ten observations is possible.

26. KKKK(80)--This array controls the printing section of the BUG subroutine. If KKKK(k) is one and BUG(k) is called then debug information will be printed. If KKKK(k) is zero no information will be written.

27. LY--The last year of the property account is represented by this variable.

28. N--The variable is used in the INPUT subroutine denoting the number of changes of data per card. It is last used in counting the number of variations in the flowthrough distribution and retains this value.

29. NPTS--The number of observation points in the property account is fixed by the variable.

30. NTR--This variable represents the number of simulated runs per property account.

31. ONE--ONE is set equal to numeral one because the author kept mixing up 1 and I on the keypunch.

32. PA(51,51)--This array holds the property account records. Each row is a vintage account of the property placed in service that year. PA(r,c) represents the amount of property remaining at time c from that which was put in service at time r. The information is printed by the PRNNT subroutine.

33. PLANT(50)--The quantity of property in service is stored in this array. The information is utilized in calculating the depreciation charge and is printed by the PRNNT subroutine.

34. R1 and R2--The range of the years to be printed from the property account. These values are calculated in the PAGE2 subroutine and utilized in the PRNNT subroutine.

35. RAN--RAN is a two digit random number generated by the RANDOM subroutine.

36. RD--The year in which the simulation being conducted on the property account is denoted by this variable.

37. REN(100,10)--This array collects the renewal data which will be arranged by the HISAGN subroutine. REN(r,c) represents the number of renewals in the r simulation at the c observation point.

38. RENH(11,10)--The histogram of renewals is stored in this array. RENH(r,c) is the percent of renewals in the r grouping on observation point c.

39. RENTOT(50) --the accumulated renewals are held in this array. RENTOT(k) denotes the total renewals at the k time period. The information is used in calculating the depreciation reserve and is printed by the PRNNT subroutine.

40. RES(100,10)--This array is utilized in the same manner as REN except that depreciation reserve information is collected.

41. RESH(11,10)--This array is utilized in the same manner as RENH(11,10) except it is a histogram of depreciation reserve values,

42. RM(20,2)--The flow though variation distribution is stored in this array. The variation and probability of occurrence are represented by columns one and two respectively.

43. RPER--The variable denotes the random flowthrough variation.

44. RR(31)--The adjusted life distribution of the property is held in this array.

45. RRPER--This variable represents the flowthrough variation due to change in average service life from one year to another.

46. SC(400)--This array holds the Iowa Survivor Curve that is originally read off the tape. SC(k) denotes the percent surviving in year k. The average service life is one hundred years.

47. SEED--This variable is the random number generator seed and is initially set equal to IPA.

48. SM(10)--This array is used in the HISAGN subroutine and is equivalent to SMRES(10) or SMREN(10).

50. SMRES(10)--The array serves the same purpose as SMREN(10) except it holds the smallest value of the depreciation reserve in the k observation point.

51. SUCU--The Iowa Survivor Curve number is denoted by this variable.

52. SUCUX--The Iowa Survivor Curve number in the previous year is held by this variable.

53. TAPE--The input number of the magnetic tape equals ten and is denoted by this variable.

54. U(51,7)--The operational data of the property account

are recorded in this array. The year, major property placements, renewals, growth, total property to be placed in service, placement adjustment, and the total retirements are found in columns one through seven respectively.

55. V(31)--This array holds the distribution density of the Iowa Survivor Curve before adjustment is made.

56. WB(3,4)--This array contains the information required in the P matrix. It was used to save computer storage, otherwise a fifty-one by fifty matrix would be required.

57. Y(51,3)--Information of the second through fifth data cards is held.

APPENDIX II:

MISCELLANEOUS FORMULA DEVELOPMENT

Section A

$$\sum xf(x)=1f(1)+2f(2)+3f(3)+.......+nf(n)$$

$$ASL=f(1)/4+1f(2)+2f(3)+...+(n-1)f(n)$$

$$\sum xf(x)-ASL=-f(1)/4+f(1)+f(2)+...+f(n)$$

$$=1-f(1)/4$$

Section B

$$g(x)=f(x+1)/(1-f(1))$$

$$\sum xg(x)=1g(1)+2g(2)+3g(3)+...+(n-1)g(n-1)$$

$$=(1f(2)+2f(3)+3f(4)+...+(n-1)f(n))/(1-f(1))$$

The above expression is reasonably close to

$$ASL=f(1)/4+1f(2)+2f(3)+3f(4)+...+(n-1)f(n)$$

Section C

Let: ASL=average service life of property presently being placed in service.

ASLP=average service life of property when it was previously placed in service.

RR=retirement rate=1/ASL

RRP=retirement rate=1/ASLP

The percentage increase(+) or decrease(-) in retirement rate is

$$(RR-RRP)/RRP$$

Expressing the above equation in terms of average service lives it becomes

$$((1/ASL)-(1/ASLP))/(1/ASLP)=(ASLP-ASL)/ASL$$

APPENDIX III:

PROBABILITY OF MISCLASSIFICATION

Let p(i) be a uni-variate normally distributed population with mean u(i) and variance V. Let r(i) represent a region of classification such that if an observation is in r(i), it is classified to be in population p(i). Let q(i) denote the priori probability that an observation is from p(i). Finally, let c(i|j) represent the cost of classifying the observation in i when it is in j.

From Anderson (2, p. 134) the best regions of classification for an observation z are given by:

r(1): $z[u(1)-u(2)]/V-[u(1)-u(2)][u(1)+u(2)]/2V \geq \log(k)$

r(2): $z[u(1)-u(2)]/V-[u(1)-u(2)][u(1)+u(2)]/2V < \log(k)$

where $k= q(2)c(1|2)/q(1)c(2|1)$

If the costs are identical, and the two populations equally like, then k=1, and log(k)=0. This implies the classification criteria is at the midpoint of the two means. Anderson (2, p. 137) suggests when the means and variances are estimated, that u(i) be replaced by the estimate $\bar{x}$ and V be replaced by the estimate s^2. Let c=log(k). Consider r(1):

$z > \{\log(k)+[\bar{x}(0\%)+\bar{x}(p\%)][\bar{x}(0\%)-\bar{x}(p\%)]/2s^2\}/\{[\bar{x}(0\%)-\bar{x}(p\%)]/s^2\}$

Since $\bar{x}(0\%) > \bar{x}(p\%)$, the probability of misclassification will be examined for a criteria of $z \geq \bar{x}(0\%) - x$ where x =is. For this criteria

$c=\{[\bar{x}(0\%)-x][\bar{x}(0\%)-\bar{x}(p\%)] - [\bar{x}(0\%)+\bar{x}(p\%)][\bar{x}(0\%)-\bar{x}(p\%)]/2\}/s^2$

Note that if $\bar{x}(0\%)-x$ is the midpoint, $\{\bar{x}(0\%)-\bar{x}(p\%)\}/2$, then c again equals zero.

The probabilities of misclassification $p(0\%|p\%,x)$ were based on a value x, a distance from $\bar{x}(0\%,z)$. Anderson (2, p. 135) calculated the probability of misclassification as

$$p(0\%|p\%) = k\int_b^\infty \exp(-y^2/2)\,dy$$

where $k=1/\sqrt{2\pi}$

$b=(c+a/2)/\sqrt{a}$

$a=[\bar{x}(0\%)-\bar{x}(p\%)][\bar{x}(0\%)-\bar{x}(p\%)]/s^2$

APPENDIX IV:

TABULATION OF TRIALS 1-4

Section A

Initial Placement: $10,000
Mortality Characteristics: S(3)-5

ASL Year	5	6	7
1	1000	833	714
2	2993	2498	2142
3	4821	4109	3550
4	5853	5396	4821
5	5521	5899	5687
6	4433	5441	5875
7	3928	4519	5382
8	4399	3954	4583
9	5005	4131	4016
10	5056	4704	3995
11	4718	5085	4413
12	4467	5042	4892
13	4523	4750	5116
14	4724	4502	5030
15	4815	4469	4774
16	4741	4613	4540
17	4635	4773	4455
18	4617	4822	4531
19	4673	4755	4682
20	4721	4658	4801

Section B

Pure Depreciation Reserve Values-$

Initial Placement: $10,000
Mortality Characteristics: S(3)-5
Trends Begin in Year Eleven

Trend Year	3%	5%	7%
11	4651	4606	4561
12	4352	4275	4199
13	4369	4267	4166
14	4519	4383	4247
15	4548	4371	4193
16	4421	4208	3995
17	4277	4039	3801
18	4222	3959	3696
19	4230	3934	3637
20	4220	3884	3549

Section C

Pure Depreciation Reserve Values-$

Mortality Characteristics: S(3)-5
Initial Placement: $10,000
Growth Begins in Year Two

Year	Growth 3%	7%
1	1000	1000
2	3023	3063
3	4942	5106
4	6121	6494
5	5974	6617
6	5065	5992
7	4711	5907
8	5324	6792
9	6090	7873
10	6324	8476
11	6175	8731
12	6109	9090
13	6348	9783
14	6741	10669
15	7034	11507
16	7170	12238
17	7280	12989
18	7480	13880
19	7761	14908
20	8041	15999

Section D

Pure Depreciation Reserve Values-$

Initial Placement: $10,000
Mortality Characteristics:
Years 1-9: S(3)-5
Years 10-20: R(1)-7

Year	Depreciation Rate	Reserve $	Depreciation Rate	Reserve $
10	0.200	5056	0.143	4486
11	0.200	5266	0.143	4126
12	0.200	5443	0.143	3733
13	0.200	5715	0.143	3435
14	0.200	6141	0.143	3291
15	0.200	6722	0.143	3302
16	0.200	7416	0.143	3426
17	0.200	8157	0.143	3547
18	0.200	8877	0.143	3747
19	0.200	9534	0.143	3834
20	0.200	10111	0.143	3841

Year	Depreciation Rate	Reserve $
10	0.200	5056
11	0.200	5266
12	0.143	4873
13	0.143	4575
14	0.143	4431
15	0.143	4442
16	0.143	4566
17	0.143	4737
18	0.143	4887
19	0.143	4974
20	0.143	4981

APPENDIX V:

TABULATION OF TRIAL 5

Simulated Depreciation Reserve Values-$

Initial Placement: $10,000
Mortality Characteristics: S(3)-5

Year / Account	11	12	13	14	15	16	17	18	19	20
1	4481	4275	4446	4629	4804	4742	4709	4688	4833	4936
2	5183	4930	4987	5146	5348	5319	5169	5134	5173	5226
3	4748	4575	4672	4800	4835	4780	4634	4568	4624	4571
4	4797	4503	4614	4890	4928	4876	4785	4708	4758	4804
5	4691	4525	4556	4729	4877	4927	4775	4692	4681	4735
6	4805	4571	4599	4770	4799	4744	4720	4765	4811	4791
7	4872	4637	4602	4711	4801	4698	4624	4610	4653	4692
8	5090	4781	4736	4966	5022	5051	5069	4976	5011	5058
9	4582	4438	4507	4699	4836	4763	4664	4709	4859	4844
10	4612	4388	4552	4799	4892	4826	4724	4644	4698	4806
11	4470	4377	4506	4748	4832	4701	4546	4539	4546	4599
12	4907	4773	4893	5008	5046	4938	4923	4855	4965	5106
13	4714	4422	4422	4549	4693	4645	4563	4604	4642	4679
14	4724	4504	4499	4614	4689	4738	4727	4710	4746	4779
15	4781	4539	4517	4637	4822	4842	4826	4801	4775	4752
16	4837	4655	4628	4809	5002	5061	4911	4823	4807	4920
17	4729	4607	4637	4820	4857	4735	4584	4639	4703	4756
18	4827	4525	4557	4791	4893	4783	4628	4607	4761	4878
19	4598	4390	4462	4708	4787	4650	4495	4592	4714	4827
20	4645	4414	4472	4714	4856	4732	4639	4726	4877	4766
21	4732	4508	4551	4671	4847	4794	4710	4637	4622	4602
22	4776	4554	4665	4849	4877	4811	4782	4769	4764	4745
23	4998	4682	4646	4768	4873	4836	4813	4783	4758	4739
24	4699	4463	4514	4792	4882	4822	4788	4824	4810	4855
25	4791	4571	4553	4723	4806	4753	4608	4692	4735	4782
26	4908	4605	4580	4701	4797	4818	4737	4651	4687	4788
27	4704	4544	4647	4825	4851	4727	4582	4581	4646	4754
28	4654	4447	4498	4672	4751	4688	4699	4679	4725	4826
29	4808	4694	4655	4816	4908	4804	4723	4704	4749	4733
30	4694	4455	4568	4846	4877	4756	4609	4605	4728	4779

Simulated Depreciation Reserve Values-$

Initial Placement: $10,000
Mortality Characteristics: S(3)-5
3% trend beginning in year 11

Year / Account	11	12	13	14	15	16	17	18	19	20
1	4487	4279	4254	4453	4571	4378	4232	4176	4130	4126
2	4607	4339	4396	4519	4539	4481	4443	4319	4252	4173
3	4721	4500	4492	4607	4622	4446	4323	4335	4399	4381
4	4651	4436	4432	4487	4434	4324	4157	4122	4122	4100
5	5077	4708	4616	4675	4781	4638	4574	4598	4520	4501
6	4523	4194	4283	4436	4461	4388	4234	4175	4240	4328
7	4545	4214	4234	4467	4566	4446	4288	4213	4159	4159
8	4559	4225	4347	4552	4649	4517	4289	4163	4181	4135
9	4838	4539	4628	4862	4850	4666	4458	4466	4434	4493
10	4581	4308	4274	4349	4302	4251	4136	4090	4140	4110
11	4675	4455	4463	4538	4490	4369	4352	4371	4370	4333
12	4560	4352	4356	4418	4367	4355	4299	4312	4299	4197
13	4701	4464	4519	4602	4624	4600	4563	4446	4380	4298
14	4656	4320	4424	4555	4586	4406	4207	4100	4176	4179
15	4810	4522	4447	4560	4650	4553	4363	4235	4227	4325
16	4601	4270	4304	4454	4542	4414	4202	4084	4192	4256
17	4477	4300	4318	4458	4536	4467	4260	4198	4396	4390
18	4594	4331	4342	4471	4433	4418	4286	4232	4171	4090
19	4841	4495	4499	4698	4742	4561	4410	4407	4422	4359
20	4917	4610	4532	4597	4570	4475	4362	4369	4359	4272
21	4753	4463	4456	4585	4716	4606	4464	4451	4505	4435
22	4846	4569	4554	4619	4651	4548	4421	4305	4398	4385
23	4661	4477	4415	4477	4433	4328	4280	4236	4166	4131
24	4640	4362	4429	4628	4661	4464	4251	4145	4173	4181
25	4648	4327	4345	4472	4492	4377	4244	4287	4285	4268
26	4725	4518	4597	4654	4677	4565	4378	4330	4334	4321
27	4758	4454	4445	4634	4768	4592	4383	4259	4273	4344
28	4749	4453	4443	4517	4549	4503	4431	4365	4291	4204
29	4885	4520	4450	4581	4567	4469	4340	4279	4276	4263
30	4842	4495	4551	4637	4736	4722	4511	4439	4437	4375

Simulated Depreciation Reserve Values-$

Initial Placement: $10,000
Mortality Characteristics: S(3)-5
5% trend beginning in year 11

Year / Account	11	12	13	14	15	16	17	18	19	20
1	4600	4353	4385	4421	4393	4234	4179	4164	4186	4212
2	4690	4301	4336	4382	4430	4382	4149	4052	4011	3907
3	4542	4246	4236	4380	4353	4195	4093	4106	4013	3888
4	4560	4356	4401	4501	4468	4293	4071	4007	4047	4058
5	4464	4227	4273	4359	4254	4026	3885	3776	3756	3697
6	4910	4665	4550	4574	4497	4425	4229	4210	4263	4139
7	4544	4174	4113	4279	4275	4122	4008	3915	3882	3892
8	4494	4139	4075	4167	4252	4170	4002	3895	3845	3739
9	4668	4283	4208	4406	4469	4383	4196	4023	3927	3894
10	4859	4451	4415	4613	4724	4512	4260	4097	4139	4064
11	4657	4327	4306	4347	4263	4052	3972	3909	3879	3872
12	4647	4328	4360	4469	4400	4176	4013	4043	4089	4034
13	4496	4207	4257	4350	4259	4034	3982	3978	4014	3943
14	4592	4275	4315	4477	4403	4236	4163	4181	4098	3971
15	4686	4368	4288	4477	4470	4321	4089	3941	3859	3885
16	4677	4433	4406	4436	4410	4321	4171	4034	3932	3873
17	4702	4315	4247	4360	4360	4275	4103	3949	3853	3906
18	4512	4201	4282	4389	4370	4261	4145	4002	3971	4015
19	4559	4201	4186	4278	4261	4052	3995	3913	3880	3760
20	4794	4411	4329	4433	4497	4357	4124	4026	4052	4017
21	4729	4461	4425	4456	4373	4166	4029	3968	3999	3937
22	4628	4324	4296	4386	4371	4325	4160	4067	4080	4084
23	4878	4484	4434	4516	4452	4320	4168	4084	3989	4035
24	4561	4308	4383	4429	4339	4109	3961	3967	3952	3827
25	4741	4359	4337	4496	4556	4404	4161	4005	3923	3896
26	4678	4374	4306	4405	4391	4241	4083	4000	4064	4013
27	4795	4431	4450	4625	4555	4405	4237	4095	4071	4028
28	4579	4323	4291	4383	4427	4377	4145	3989	3890	3850
29	4691	4368	4328	4415	4342	4199	4107	4086	4049	3922
30	4759	4433	4331	4356	4346	4155	4014	3935	3897	3842

Simulated Depreciation Reserve Values-$

Initial Placement: $10,000
Mortality Characteristics: S(3)-5
7% trend beginning in year 11

Account \ Year	11	12	13	14	15	16	17	18	19	20
1	4565	4263	4148	4258	4140	3957	3778	3617	3552	3463
2	4535	4246	4188	4246	4250	4165	3907	3725	3593	3516
3	4647	4290	4225	4278	4166	3988	3869	3821	3751	3586
4	4714	4355	4227	4220	4170	3945	3776	3669	3599	3507
5	4932	4506	4374	4495	4411	4302	4203	4014	3936	3845
6	4427	4163	4143	4213	4203	3997	3804	3758	3793	3641
7	4456	4121	4186	4325	4268	4052	3841	3682	3636	3615
8	4315	4097	4135	4265	4189	3905	3652	3569	3472	3389
9	4750	4487	4514	4526	4404	4143	4025	3885	3888	3888
10	4558	4154	4062	4070	4077	3901	3718	3659	3584	3491
11	4566	4227	4126	4119	4054	3983	3871	3753	3660	3552
12	4622	4259	4145	4146	4195	4084	3956	3824	3673	3520
13	4677	4366	4252	4314	4362	4282	4023	3832	3695	3680
14	4572	4322	4267	4336	4218	3961	3723	3695	3647	3556
15	4670	4254	4195	4316	4279	4034	3776	3666	3713	3702
16	4443	4123	4102	4229	4165	3900	3649	3649	3662	3639
17	4484	4131	4094	4214	4214	3956	3760	3746	3787	3642
18	4577	4238	4189	4191	4234	4050	3861	3686	3555	3413
19	4620	4285	4300	4371	4253	4054	3925	3828	3710	3550
20	4840	4409	4285	4294	4258	4092	3969	3850	3713	3558
21	4543	4195	4154	4317	4266	4068	3923	3865	3746	3654
22	4636	4300	4191	4242	4188	4012	3773	3760	3692	3602
23	4748	4319	4201	4201	4155	4051	3873	3694	3611	3581
24	4548	4266	4276	4342	4216	3953	3715	3634	3588	3559
25	4499	4178	4131	4190	4131	3945	3858	3747	3675	3633
26	4651	4400	4272	4315	4255	4018	3847	3745	3678	3527
27	4537	4187	4204	4373	4260	3990	3732	3641	3663	3585
28	4715	4350	4214	4309	4324	4205	4007	3821	3686	3547
29	4716	4300	4253	4272	4229	4046	3858	3748	3683	3693
30	4679	4404	4300	4416	4462	4211	4012	3894	3775	3698

APPENDIX VI:

SUMMARY OF TRIAL 5

Simulated Depreciation Reserve Variances

Trend Year	0%	3%	5%	7%	Pooled
11	24,297	19,703	13,788	16,330	18,529
12	18,990	15,700	11,881	11,368	14,484
13	14,737	11,537	9,772	8,297	11,085
14	14,791	10,711	9,254	9,843	11,149
15	14,519	16,378	11,173	8,527	12,649
16	19,030	12,435	16,497	11,389	14,837
17	22,060	12,726	8,635	15,662	14,770
18	15,885	16,176	8,666	9,779	12,626
19	15,794	13,202	12,701	9,346	12,760
20	19,707	13,849	13,546	11,644	14,686

Simulated Depreciation Reserve Means
$

Trend Year	0%	3%	5%	7%
11	4762	4698	4657	4609
12	4546	4417	4338	4274
13	4592	4429	4319	4213
14	4774	4553	4419	4281
15	4870	4586	4399	4233
16	4812	4478	4211	4042
17	4726	4338	4097	3857
18	4711	4284	4014	3730
19	4762	4287	3987	3681
20	4808	4271	3940	3594

Pure Depreciation Reserve Values
$

Trend Year	0%	3%	5%	7%
11	4718	4651	4606	4561
12	4468	4352	4275	4199
13	4523	4369	4267	4166
14	4724	4519	4383	4247
15	4815	4548	4371	4193
16	4741	4421	4208	3995
17	4635	4277	4039	3801
18	4617	4222	3959	3696
19	4673	4230	3934	3637
20	4721	4220	3884	3549

APPENDIX VII:

PROBABILITY OF MISCLASSIFICATION DATA

Criteria is x standard deviations from $\bar{x}$(0%,z)

P(0%|3%,x)

year	x 0.5	1.0	1.5	2.0	2.5	3.0	mid point
11	0.512	0.702	0.848	0.936	0.978	0.994	0.405
12	0.284	0.472	0.666	0.824	0.924	0.973	0.295
13	0.147	0.291	0.430	0.674	0.829	0.926	0.221
14	0.056	0.138	0.278	0.464	0.659	0.819	0.146
15	0.021	0.063	0.154	0.298	0.488	0.681	0.104
16	0.013	0.041	0.107	0.230	0.405	0.599	0.086
17	0.004	0.014	0.046	0.106	0.245	0.425	0.056
18	0.000	0.003	0.011	0.036	0.097	0.212	0.029
19	0.000	0.000	0.003	0.014	0.044	0.115	0.018
20	0.000	0.000	0.002	0.008	0.027	0.077	0.014

P(0%|5%,x)

Year	x 0.5	1.0	1.5	2.0	2.5	3.0	mid point
11	0.394	0.591	0.768	0.891	0.958	0.987	0.348
12	0.111	0.236	0.409	0.606	0.779	0.898	0.194
13	0.018	0.056	0.138	0.278	0.464	0.659	0.099
14	0.002	0.010	0.033	0.092	0.206	0.378	0.049
15	0.000	0.000	0.004	0.014	0.046	0.117	0.018
16	0.000	0.000	0.000	0.005	0.017	0.054	0.011
17	0.000	0.000	0.000	0.000	0.004	0.015	0.005
18	0.000	0.000	0.000	0.000	0.000	0.000	0.000
19	0.000	0.000	0.000	0.000	0.000	0.000	0.000
20	0.000	0.000	0.000	0.000	0.000	0.000	0.000

P(0%|7%,x)

Year	x 0.5	1.0	1.5	2.0	2.5	3.0	mid point
11	0.268	0.452	0.648	0.811	0.916	0.970	0.288
12	0.039	0.104	0.224	0.397	0.595	0.770	0.129
13	0.000	0.005	0.018	0.055	0.136	0.274	0.036
14	0.000	0.000	0.000	0.004	0.015	0.046	0.000
15	0.000	0.000	0.000	0.000	0.002	0.004	0.000
16	0.000	0.000	0.000	0.000	0.000	0.000	0.000
17	0.000	0.000	0.000	0.000	0.000	0.000	0.000
18	0.000	0.000	0.000	0.000	0.000	0.000	0.000
19	0.000	0.000	0.000	0.000	0.000	0.000	0.000
20	0.000	0.000	0.000	0.000	0.000	0.000	0.000